Bilateral and Multilateral Estimates of the Relative Purchasing Power of the Canadian and US Dollars

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This paper compares the relative purchasing power of the Canadian and US dollars in 1985, 1990 and 1993 using data from the International Comparison Programme (ICP) and OECD. It is shown that uncertainty over the Canadian/US dollar purchasing power parity (PPP) is largely attributable to the fact that most estimates have been drawn from multilateral comparisons which are inherently less reliable. In contrast, the results from bilateral comparisons are more consistent, and suggest that the purchasing power of the US dollar, between 1985 and 1993, remained reasonably close to 1.25 Canadian dollars. (*JEL* C43, E31, O51)

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1. Introduction

It is perhaps surprising to anyone familiar with the superlative index number literature that there is any uncertainty about the relative purchasing power of the Canadian and US dollars. This is because a consensus has emerged in the index number literature that bilateral comparisons should be made using superlative indexes. Superlative price indexes have the attractive property of coinciding exactly with the underlying cost of living index if preferences are described by a specific flexible functional form.[1] The choice of superlative formula is not particularly important, since they should all give similar answers. Hence, as long as a superlative formula is used, there should be very little uncertainty about the Canadian/US dollar purchasing power parity (PPP).

However, most published estimates of the Canadian/US dollar PPP, such as those from the International Comparisons Programme (ICP), OECD, IMF, World Bank and UN, are drawn from multilateral comparisons. Superlative indexes cannot be used in such comparisons since they are intransitive and hence internally inconsistent in a multilateral context. This problem of transitivity makes multilateral comparisons more complicated. In fact, no consensus has emerged as yet in the literature with regard to how such comparisons should be made. A large number of multilateral methods have been proposed, many of which are surveyed in Hill (1997).

Estimates of the Canadian/US dollar PPP vary quite significantly across multilateral comparisons for two reasons. First, different organizations use different multilateral formulae. In particular, the ICP uses the Geary-Khamis method, the OECD uses both the Geary-Khamis and Eltetö-Köves-Szulc (EKS) methods, while Eurostat uses just the EKS method. The World Bank currently uses Geary-Khamis, although recently it has been debating whether to switch to a method proposed by Iklé (1972) (see Dikhanov,

[1] This terminology was introduced by Diewert (1976). The most widely used superlative indexes are Fisher and Törnqvist, both of which are defined in section 2. Fisher and Törnqvist are exact, respectively, for the homogenous quadratic and translog flexible functional forms.

1994).[2] Second, in a multilateral comparison, the resulting PPP between the US and Canada depends not only on the price and expenditure data of these two countries, but also on the data of all the other countries in the comparison. The fact that the ICP and OECD comparisons cover very different groupings of countries causes their results to differ.

This paper considers two different data sets. The first is an ICP data set consisting of price and expenditure data for 139 headings over 64 countries in 1985. The second is an OECD data set consisting of price and expenditure data for 198 headings for the 24 member countries in 1990 and 1993. Upper and lower bounds on the Canadian/US dollar PPP are computed for both bilateral and multilateral methods for each data set. The multilateral bounds, in general, far exceed the bilateral bounds. For example, in a bilateral comparison based on the 1985 ICP data set, depending on the choice of bilateral formula, the purchasing power of a US dollar lies somewhere between 1.22 and 1.25 Canadian dollars. In contrast, the corresponding multilateral estimates range between 1.05 and 1.30. Interestingly, the exchange rate in 1985 was 1.37, which lies outside the multilateral bounds. This result illustrates how misleading exchange rates can be as a measure of relative purchasing power.

The empirical results obtained in this paper emphasize the importance of using bilateral index number methods for making bilateral comparisons. Multilateral methods should only be resorted to when one wishes to simultaneously compare the relative purchasing power of three or more countries.

2. Notation and Definitions

The set of countries is indexed by $k = 1, \ldots, K$. The price and quantity data (p_{ki}, q_{ki}) of each country are defined over the same set of goods and services indexed by

[2]All these multilateral methods are defined in section 2.

$i = 1, \ldots, N$.

(i) Bilateral Price and Quantity Indexes

Let P_{bk} and Q_{bk} denote respectively a bilateral price and quantity index between countries b and k, where b denotes the base country. Four important bilateral formulae are Paasche, Laspeyres, Fisher and Törnqvist. These indexes are defined below:

$$\text{Paasche}: \quad P^P_{bk} = \frac{\sum_{i=1}^N p_{ki}q_{ki}}{\sum_{i=1}^N p_{bi}q_{ki}} \qquad Q^P_{bk} = \frac{\sum_{i=1}^N p_{ki}q_{ki}}{\sum_{i=1}^N p_{ki}q_{bi}} \tag{1}$$

$$\text{Laspeyres}: \quad P^L_{bk} = \frac{\sum_{i=1}^N p_{ki}q_{bi}}{\sum_{i=1}^N p_{bi}q_{bi}} \qquad Q^L_{bk} = \frac{\sum_{i=1}^N p_{bi}q_{ki}}{\sum_{i=1}^N p_{bi}q_{bi}} \tag{2}$$

$$\text{Fisher}: \quad P^F_{bk} = \sqrt{P^P_{bk}P^L_{bk}} \qquad Q^F_{bk} = \sqrt{Q^P_{bk}Q^L_{bk}} \tag{3}$$

$$\text{Törnqvist}: \quad P^T_{bk} = \prod_{i=1}^N \left(\frac{p_{ki}}{p_{bi}}\right)^{\frac{v_{bi}+v_{ki}}{2}} \qquad Q^T_{bk} = \prod_{i=1}^N \left(\frac{q_{ki}}{q_{bi}}\right)^{\frac{v_{bi}+v_{ki}}{2}} \tag{4}$$

$$\text{where} \quad v_{ki} = \frac{p_{ki}q_{ki}}{\sum_{i=1}^N p_{ki}q_{ki}}.$$

All four bilateral price and quantity index formulae are intransitive, and hence internally inconsistent in a multilateral comparison. Fisher and Törnqvist are superlative, while Paasche and Laspeyres are not. However, the latter are still useful benchmarks, since they provide lower and upper bounds, respectively, on the true PPP.[3] Furthermore, superlative indexes almost always lie between Paasche and Laspeyres. Indeed Fisher by construction must do so, since it is the geometric mean of Paasche and Laspeyres. Hence Paasche and Laspeyres provide useful bounds on both the true PPP and the sensitivity of the results to the choice of bilateral formula.

[3]Strictly, this is only true when preferences are homothetic, since in general the true PPP depends on the utility level of the comparison, and Paasche and Laspeyres provide bounds on the true PPP at different utility levels (see Samuelson and Swamy, 1974).

(ii) Multilateral Methods

A multilateral (transitive) PPP and quantity index for country k are denoted by P_k and Q_k respectively.

(a) Average Price Methods

Average price methods calculate the PPP and quantity index between countries b and k as follows:

$$\frac{P_k}{P_b} = \frac{P^P_{Xk}}{P^P_{Xb}}, \quad \frac{Q_k}{Q_b} = \frac{Q^L_{Xk}}{Q^L_{Xb}}. \tag{5}$$

P^P_{Xk} is the Paasche PPP defined in (1), while Q^L_{Xk} is the Laspeyres quantity index defined in (2). Alternatively, (5) may be written thus:

$$\frac{P_k}{P_b} = \frac{\sum_{i=1}^N p_{ki}q_{ki}}{\sum_{i=1}^N p_{Xi}q_{ki}} \frac{\sum_{i=1}^N p_{Xi}q_{bi}}{\sum_{i=1}^N p_{bi}q_{bi}},$$

$$\frac{Q_k}{Q_b} = \frac{\sum_{i=1}^N p_{Xi}q_{ki}}{\sum_{i=1}^N p_{Xi}q_{Xi}} \frac{\sum_{i=1}^N p_{Xi}q_{Xi}}{\sum_{i=1}^N p_{Xi}q_{bi}} = \frac{\sum_{i=1}^N p_{Xi}q_{ki}}{\sum_{i=1}^N p_{Xi}q_{bi}}. \tag{6}$$

From (6) it can be seen that average price methods differ only in how they define the price vector p_X. Country X may be one of the countries in the set, or alternatively an artificial country whose price vector is constructed by taking some average of the price vectors of the countries in the comparison.

The most widely used average price PPP method is Geary (1958)-Khamis (1972).[4] The Geary-Khamis reference price vector p_X and Paasche PPPs P^P_{Xk} are obtained by solving the system of $N+K$ simultaneous equations in (7) and (8).[5]

$$p_{Xi} = \sum_{j=1}^K \left(\frac{q_{ji}}{\sum_{r=1}^K q_{ri}} \frac{p_{ji}}{P^P_{Xj}} \right) \quad \forall i = 1, \ldots, N \tag{7}$$

[4]In particular, it is used to construct the Penn World Table. A useful reference on the ICP and Penn World Table is Summers and Heston (1991).

[5]Khamis (1972) proves existence and uniqueness for the Geary-Khamis system.

$$P_k = P^P_{Xk} = \frac{\sum_{i=1}^{N} p_{ki} q_{ki}}{\sum_{i=1}^{N} p_{Xi} q_{ki}} \quad \forall k = 1, \ldots, K \tag{8}$$

Geary-Khamis gives greater weight to the price vectors of larger countries when determining the reference price vector, p_X.

Two equally weighted variants on the Geary-Khamis reference price formula are the following:

$$p_{Xi} = \sum_{j=1}^{K} \left(\frac{v_{ji}}{\sum_{r=1}^{K} v_{ri}} \frac{p_{ji}}{P^P_{Xj}} \right) \quad \forall i = 1, \ldots, N, \tag{9}$$

$$p_{Xi} = \sum_{j=1}^{K} \left(\frac{q_{ji}/Q^L_{Xj}}{\sum_{r=1}^{K} (q_{ri}/Q^L_{Xr})} \frac{p_{ji}}{P^P_{Xj}} \right) \quad \forall i = 1, \ldots, N. \tag{10}$$

In (9), v_{ji} denotes the share of good i in the expenditure of country j as defined in (4). The formula in (10) was first proposed by Iklé (1972), and has more recently been advocated by Dikhanov (1994). However, the formula in (9) has the advantage over Iklé of being algebraically simpler and easier to compute. In practice, all equally weighted average price methods give similar results, and hence it does not matter much which of these two methods is used.

By contrast, a fixed base average price method simply uses the price vector of one country (say country k) as the reference price vector, i.e.:

$$p_{Xi} = p_{ki} \quad \forall i = 1, \ldots, N. \tag{11}$$

A fixed base method is the limiting case of a weighted average price method in the sense that it gives all weight in the average price formula to the price vector of one country.

One particularly attractive feature of average price PPP methods is that their quantity indexes are additive over different levels of aggregation when measured in value terms. This additivity property is extremely useful if international comparisons are required at various levels of aggregation, as for example in a national accounts comparison. However, using a single reference price vector to compare the purchasing power of currencies introduces substitution bias. As a result, average price methods tend to systematically overestimate the purchasing power of countries whose relative prices differ substantially from the reference prices used in the comparison. This tendency is sometimes referred to as the *Gerschenkron effect*, see Gerschenkron (1951). Nuxoll (1994) and Hill (1998) provide empirical evidence of substitution bias in the results of average price PPP methods.

(b) Average Basket Methods

Average basket methods calculate the PPP and quantity index between countries b and k as follows:

$$\frac{P_k}{P_b} = \frac{P^L_{Xk}}{P^L_{Xb}}, \quad \frac{Q_k}{Q_b} = \frac{Q^P_{Xk}}{Q^P_{Xb}}. \tag{12}$$

P^L_{Xk} is the Laspeyres PPP defined in (2), while Q^P_{Xk} is the Paasche quantity index defined in (1). Alternatively, (12) may be written thus:

$$\frac{P_k}{P_b} = \frac{\sum_{i=1}^N p_{ki}q_{Xi}}{\sum_{i=1}^N p_{Xi}q_{Xi}} \frac{\sum_{i=1}^N p_{Xi}q_{Xi}}{\sum_{i=1}^N p_{bi}q_{Xi}} = \frac{\sum_{i=1}^N p_{ki}q_{Xi}}{\sum_{i=1}^N p_{bi}q_{Xi}},$$

$$\frac{Q_k}{Q_b} = \frac{\sum_{i=1}^N p_{ki}q_{ki}}{\sum_{i=1}^N p_{ki}q_{Xi}} \frac{\sum_{i=1}^N p_{bi}q_{Xi}}{\sum_{i=1}^N p_{bi}q_{bi}}. \tag{13}$$

From (13) it can be seen that average basket methods differ only in how they define the quantity vector q_X. Again, country X may be one of the countries in the set, or alternatively an artificial country whose quantity vector is constructed by taking some average of the quantity vectors of the countries in the comparison. Three natural formulae for q_X are, respectively, the arithmetic, geometric and harmonic means of the quantity vectors of all the countries in the set. These formulae are given below:

$$q_{Xi} = \sum_{k=1}^K q_{ki} \quad \forall i = 1, \ldots, N, \tag{14}$$

$$q_{Xi} = \prod_{k=1}^K {q_{ki}}^{1/K} \quad \forall i = 1, \ldots, N, \tag{15}$$

$$q_{Xi} = \left(\sum_{k=1}^{K} \frac{1}{q_{ki}} \right)^{-1} \quad \forall i = 1, \ldots, N. \tag{16}$$

The Average basket method that calculates q_X using (14) is usually called the ECLAC method. ECLAC stands for United Nations Economic Commission for Latin America and the Caribbean. This method was advocated by Walsh (1901), who called it Scrope's method, and by Van Ijzeren (1956). It is also discussed in Ruggles (1967) and Diewert (1993). The Average basket method based on (15) was advocated by Walsh (1901) who called it Scrope's method with geometric weights. The Average basket method based on (16) does not seem to have been advocated in the literature. A logical name for it would be the Harmonic Average basket method.[6]

Although these three methods differ only in the choice of symmetric mean formula, there is one very important difference between them. The ECLAC method, defined in (14), gives countries with larger baskets greater weight in the q_X formula. In contrast, the harmonic average basket method, defined in (16), gives countries with smaller baskets larger weight in the q_X formula. Finally, the geometric average basket method, defined in (15), gives equal weight to the baskets of all countries. Therefore, which of these formulae is to be preferred depends on the desired weighting of countries.

By contrast, a fixed base average basket method simply uses the quantity vector of one country (say country k) as the reference basket, i.e.:

$$q_{Xi} = q_{ki} \quad \forall i = 1, \ldots, N. \tag{17}$$

A fixed base method is the limiting case of a weighted average basket method in the sense that it gives all weight in the average basket formula to the basket of one country.

Average basket methods are not used as much as average price methods since they are not additive. However, like average price methods, average basket methods are also subject to substitution bias. Average basket methods tend to systematically underes-

[6] A number of other average basket methods have been proposed in the literature (see Hill, 1997).

timate the purchasing power of countries whose baskets differ substantially from the reference basket used in the comparison.

(c) The EKS Method

The Eltetö and Köves (1964) and Szulc (1964) (EKS) method (which was, in fact, first proposed by Gini in 1931) is the multilateral (transitive) method used by Eurostat to make comparisons between the member countries of the European Community.[7] The OECD currently uses both Geary-Khamis and EKS.[8] The EKS price index P_k and quantity index Q_k of country k equal, respectively, the geometric mean of the K Fisher price indexes and quantity indexes derived from comparisons between country k and each of the countries in the set. The EKS method is a natural extension of the Fisher index to multilateral comparisons. The EKS formulae are depicted in equation (18).

$$\frac{P_k}{P_b} = \left(\prod_{j=1}^{K} \frac{P^F_{jk}}{P^F_{jb}} \right)^{1/K} \qquad \frac{Q_k}{Q_b} = \left(\prod_{j=1}^{K} \frac{Q^F_{jk}}{Q^F_{jb}} \right)^{1/K} \tag{18}$$

EKS quantity indexes are not additive. Since EKS has neither a reference price nor quantity vector, however, it is free of substitution bias. In fact, it must be free of any systematic bias, since by construction it equals a geometric mean of superlative Fisher indexes.[9]

3. Previous Estimates of the Canadian/US PPP

This section looks at estimates of the Canadian/US PPP in 1985, 1990 and 1993 obtained by the OECD, Penn World Table and UN. According to the OECD (1998),

[7]See for example Eurostat (1983).

[8]See for example OECD (1995) and OECD (1996).

[9]See Hill (1997) for a more detailed taxonomy of multilateral PPP methods.

one US dollar was worth, respectively, 1.29, 1.30 and 1.26 Canadian dollars in 1985, 1990 and 1993. These results are drawn from a multilateral comparison of all 24 member countries. The OECD has also made bilateral comparisons between Canada and the US in 1985 and 1990 (see OECD, 1993). The PPPs in this case were 1.24 and 1.23, respectively. Meanwhile, the Penn World Table provides estimates for 1985, 1990 and 1992 of 1.21, 1.21 and 1.18, respectively.[10] Finally, the UN (1993) provides an estimate of 1.27 for 1985. From all these sources we can reasonably conclude that the Canadian/US PPP lies somewhere between 1.2 and 1.3, and has not changed much over this period. However, from the information given above, it is difficult to be more precise.

4. Measuring the Sensitivity of the Canadian/US PPP to the Choice of Formula and Data Set

This section computes bilateral and multilateral estimates of the Canadian/US dollar PPP in 1985, 1990 and 1993. The 1985 data set was constructed by the ICP, while the 1990 and 1993 data sets are from the OECD. The sensitivity of the multilateral results to the choice of formula and the set of countries is analyzed, and bilateral methods are used to provide more accurate estimates of the true PPP.

(i) The ICP Data Set

The ICP data set consists of price and expenditure data for 139 headings over 64 countries in 1985. Here we compute the Canadian/US dollar PPP for 66 average price methods. The average price methods considered are Geary-Khamis, defined in (7) and (8), equally weighted Geary Khamis, defined in (9), and the 64 fixed base average price

[10]These figures are drawn from the Penn World Table internet website, http://www.nber.org/pwt56.html.

methods obtained by using each country's price vector as the reference price vector in (11). Similarly, the Canadian/US dollar PPP is also computed for 64 fixed base average basket methods, obtained by using each country's quantity vector as the reference basket in (17). The final multilateral method considered is EKS. In addition the Fisher PPP between the US and Canada is computed. The corresponding Paasche and Laspeyres PPPs can be obtained from the fixed base average price and quantity results. The Canadian/US Paasche PPP is obtained from the fixed base average price method using the US price vector or from the fixed base average basket method using Canada's basket. Conversely, the Canadian/US Laspeyres PPP is obtained from the fixed base average price method using Canada's price vector or from the fixed base average basket method using the US basket.

The results for the average price methods in Table 1 show the amount of Canadian dollars that have the same purchasing power as one US dollar.[11] The first 64 results correspond to the fixed base average price methods with each country's price vector used, in turn, as the reference price vector. GK and EqGK denote, respectively, the Geary-Khamis and equally weighted Geary-Khamis result. Min and Max denote the minimum and maximum average price PPPs. Finally, the last three results correspond

[11] The country codes in Table 1 are: DEU - Germany, FRA - France, ITA - Italy, NLD - the Netherlands, BEL - Belgium, LUX - Luxembourg, GBR - Great Britain, IRL - Ireland, DNK - Denmark, GRC - Greece, ESP - Spain, PRT - Portugal, AUT - Austria, FIN - Finland, NOR - Norway, SWE - Sweden, AUS - Australia, NZL - New Zealand, JPN - Japan, CAN - Canada, USA - United States, TUR - Turkey, HKG - Hong Kong, KOR - South Korea, THA - Thailand, IND - India, IRN - Iran, LKA - Sri Lanka, PAK - Pakistan, PHL - the Philippines, BWA - Botswana, EGY - Egypt, ETH - Ethiopia, KEN - Kenya, MWI - Malawi, MUS - Mauritius, NGA - Nigeria, SLE - Sierra Leone, SWZ - Swaziland, TZA - Tanzania, ZMB - Zambia, ZWE - Zimbabwe, BEN - Benin, CMR - Cameroon, COG - Congo, CIV - Ivory Coast, MDG - Madagascar, MLI - Mali, MAR - Morrocco, RWA - Rwanda, SEN - Senegal, TUN - Tunisia, POL - Poland, HUN - Hungary, YUG - Yugoslavia, BHS - Bahamas, BRB - Barbados, GRD - Grenada, JAM - Jamaica, LCA - St. Lucia, SUR - Suriname, TTO - Trinidad and Tobago, BGD - Bangaldesh, NPL - Nepal.

to EKS, Fisher and exchange rates.

The Paasche-Laspeyres spread in Table 1 is from 1.22 to 1.25, and Fisher equals 1.23. If Paasche and Laspeyres are viewed as lower and upper bounds on the true underlying PPP, then from a bilateral perspective there is very little uncertainty about the Canadian/US PPP. It lies somewhere between 1.22 and 1.25. By contrast, depending on the choice of reference price vector, the Canadian/US PPP ranges between 1.05 and 1.30. Hence, from a multilateral perspective, there is much greater uncertainty. It is also interesting that the exchange rate in 1985 is 1.37, which lies outside the average price PPP upper and lower bounds.

Insert Table 1 Here

The first 64 results in Table 2 correspond to the fixed base average basket methods with each country's basket used, in turn, as the reference quantity vector. Min and Max denote the minimum and maximum average basket PPPs between Canada and the US. Finally, the last three results, again, correspond to EKS, Fisher and exchange rates.

The Paasche-Laspeyres spread and Fisher index, by construction, are the same as in Table 1, i.e., Paasche = 1.22, Laspeyres = 1.25, and Fisher = 1.23. Now, depending on the choice of reference basket, the Canadian/US PPP ranges between 1.14 and 1.33. Although this range is smaller than for the average price methods, it still far exceeds the Paasche-Laspeyres spread. Again, the 1985 exchange rate lies outside the average basket upper and lower bounds.

Insert Table 2 Here

If we restrict attention to the 22 OECD countries in the data set, the range of the fixed base average price method is reduced only slightly from (1.05,1.30) to (1.10,1.30).[12] For fixed base average basket methods, however, the reduction is larger, from (1.14,1.33) to (1.16,1.25).

(ii) The OECD Data Set

The OECD data set consists of price and expenditure data over 198 headings for the 24 member countries in 1990 and 1993. Table 3 shows the Canadian/US dollar PPP in 1990 for 26 average price methods and 24 average basket methods. In addition, the EKS, Fisher and exchange rate conversions are also included. Table 4 provides similar results for 1993. In Table 4, however, results are not available for the Geary-Khamis and equally weighted Geary-Khamis methods.

Insert Table 3 Here

Insert Table 4 Here

The Paasche-Laspeyres spread in Table 3 is from 1.22 to 1.32, and Fisher equals 1.27. This Paasche-Laspeyres spread is somewhat larger than the one obtained for the ICP data set, thus suggesting that there is greater uncertainty about the true PPP in 1990. In fact, the same bounds are obtained for average basket methods, although for average price methods, the upper bound is higher. Therefore, for this data set, the bilateral bounds are not much tighter than the multilateral bounds. The exchange rate again lies outside the multilateral bounds. However, the exchange rate now lies below the lower bound, while for the 1985 ICP data set, it lay above the upper bound!

The Paasche-Laspeyres spread in Table 4 is from 1.21 to 1.29, and Fisher equals

[12] The two missing OECD countries in the ICP data set are Switzerland and Iceland. In Tables 3 and 4, below, the codes for these countries are CHE and ICE, respectively.

1.25. The average price multilateral bounds are 1.21 and 1.37. Hence, although the average price lower bound is the same as the bilateral lower bound, the upper bound is significantly larger. In contrast, the average basket upper bound is the same as the bilateral upper bound, although its lower bound is slightly smaller. The 1993 exchange rate lies on the bilateral and average basket upper bound, but in the middle of the average basket range.

The smaller discrepancies between the bilateral and multilateral results for the OECD data set are probably attributable to the fact that the OECD countries are much less heterogenous than the countries in the ICP data set. In other words, as the set of countries becomes more diverse, the bounds on the multilateral Canadian/US PPP should rise more than the bounds on the bilateral Canadian/US PPP. This is because as the set of countries becomes more heterogenous, more and more of the price and quantity vectors in the comparison are uncharacteristic of both the Canadian and US price and quantity vectors, thus increasing the variability of the results. If the set of countries is very heterogenous, this will also tend to undermine the quality of the basic headings from which the PPPs are constructed, since it then becomes very hard to construct headings that are representative of most countries in the comparison. This may explain why restricting the comparison to just the OECD countries for the ICP data set does not significantly reduce the multilateral bounds on the Canadian/US PPP.

(iii) Which PPP?

Thus far it has been assumed that the best measure of relative purchasing power is provided by a superlative index such as Fisher. However, this is not always true. The appropriate choice of formula depends on the user's purpose. For Canadian visitors to the United States it is appropriate to compare purchasing power using the Canadian basket as weights, i.e., to use the Paasche formula, if the US is treated as the base country. Conversely, for American visitors to Canada, the Laspeyres formula should be used. Only if we wish to compare the purchasing power of the Canadian and US dollars from the perspective of a neutral third party is the Fisher index appropriate.

(iv) Changes in the Canadian/US PPP Over Time

The most reliable estimates of the Canadian/US PPP in the results above (from the perspective of a neutral third party) are the Fisher PPPs. Accordingly, the results suggest that the relative purchasing power of the US dollar rose slightly between 1985 and 1990 from 1.23 to 1.27 Canadian dollars, but then fell between 1990 and 1993 back to 1.25 Canadian dollars. These movements are sufficiently small that they may be attributable to measurement error. Alternatively, they may be the result of differing rates of inflation during this period in Canada and the US. Between 1985 and 1990, the Canadian and US GDP deflators rose, respectively, by 20.6 and 19.2 percent. This is consistent with the observed increase in the relative purchasing power of the US dollar between 1985 and 1990. Between 1990 and 1993, the Canadian and US GDP deflators rose, respectively, by 7.4 and 11.4 percent. This is also consistent with the observed decline in the relative purchasing power of the US dollar between 1990 and 1993. Over the whole period from 1985 to 1993, however, the US GDP deflator rose by 32.8 percent compared with 29.5 percent for Canada. This contradicts the observed trend in relative purchasing power between 1985 and 1993, which entailed a slight increase in the purchasing power of the US dollar from 1.23 to 1.25 Canadian dollars. Given, the small magnitudes of all these changes, it seems reasonable to conclude that the relative purchasing power of the US dollar remained approximately fixed at 1.25 Canadian dollars between 1985 and 1993.

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TABLE 1.— ICP DATA 1985 - AVERAGE PRICE METHODS

(CAN/US PPPs)

DEU	1.27	JPN	1.11	NGA	1.24	YUG	1.21
FRA	1.19	CAN	1.25	SLE	1.05	BHS	1.12
ITA	1.19	USA	1.22	SWZ	1.15	BRB	1.14
NLD	1.19	TUR	1.10	TZA	1.24	GRD	1.12
BEL	1.25	HKG	1.08	ZMB	1.18	JAM	1.12
LUX	1.24	KOR	1.10	ZWE	1.19	LCA	1.13
GBR	1.18	THA	1.19	BEN	1.17	SUR	1.16
IRL	1.25	IND	1.20	CMR	1.15	TTO	1.07
DNK	1.26	IRN	1.27	COG	1.15	BGD	1.15
GRC	1.21	LKA	1.26	CIV	1.19	NPL	1.22
ESP	1.25	PAK	1.15	MDG	1.16	GK	1.16
PRT	1.25	PHL	1.16	MLI	1.15	EqGK	1.13
AUT	1.28	BWA	1.18	MAR	1.21	Min	1.05
FIN	1.29	EGY	1.22	RWA	1.19	Max	1.30
NOR	1.28	ETH	1.16	SEN	1.13	**EKS**	**1.21**
SWE	1.25	KEN	1.22	TUN	1.15	**Fish**	**1.23**
AUS	1.30	MWI	1.21	POL	1.27	**ExR**	**1.37**
NZL	1.16	MUS	1.16	HUN	1.21		

TABLE 2.— ICP DATA 1985 - AVERAGE BASKET METHODS

(CAN/US PPPs)

DEU	1.23	JPN	1.16	NGA	1.23	YUG	1.20
FRA	1.21	CAN	1.22	SLE	1.24	BHS	1.33
ITA	1.22	USA	1.25	SWZ	1.27	BRB	1.30
NLD	1.24	TUR	1.18	TZA	1.25	GRD	1.26
BEL	1.21	HKG	1.29	ZMB	1.19	JAM	1.30
LUX	1.23	KOR	1.17	ZWE	1.27	LCA	1.25
GBR	1.22	THA	1.14	BEN	1.21	SUR	1.27
IRL	1.23	IND	1.23	CMR	1.22	TTO	1.27
DNK	1.25	IRN	1.19	COG	1.24	BGD	1.28
GRC	1.18	LKA	1.19	CIV	1.24	NPL	1.27
ESP	1.23	PAK	1.25	MDG	1.27	Min	1.14
PRT	1.21	PHL	1.29	MLI	1.23	Max	1.33
AUT	1.21	BWA	1.27	MAR	1.24	**EKS**	**1.21**
FIN	1.22	EGY	1.22	RWA	1.23	**Fish**	**1.23**
NOR	1.20	ETH	1.25	SEN	1.28	**ExR**	**1.37**
SWE	1.21	KEN	1.26	TUN	1.17		
AUS	1.23	MWI	1.25	POL	1.23		
NZL	1.22	MUS	1.21	HUN	1.21		

TABLE 3.— OECD DATA 1990 - AVERAGE PRICE AND AVERAGE BASKET METHODS (CAN/US PPPs)

Average Price PPPs				Average Basket PPPs			
DEU	1.29	FIN	1.40	DEU	1.26	FIN	1.27
FRA	1.37	ICE	1.37	FRA	1.24	ICE	1.26
ITA	1.36	NOR	1.41	ITA	1.27	NOR	1.23
NLD	1.30	SWE	1.38	NLD	1.26	SWE	1.25
BEL	1.32	TUR	1.37	BEL	1.24	TUR	1.25
LUX	1.29	AUS	1.34	LUX	1.29	AUS	1.25
GBR	1.32	NZL	1.33	GBR	1.31	NZL	1.26
IRL	1.36	JPN	1.41	IRL	1.30	JPN	1.24
DNK	1.38	CAN	1.32	DNK	1.27	CAN	1.22
GRC	1.35	USA	1.22	GRC	1.29	USA	1.32
ESP	1.35	GK	1.29	ESP	1.29	Min	1.22
PRT	1.37	EqGK	1.34	PRT	1.26	Max	1.32
AUT	1.31	Min	1.22	AUT	1.25		
CHE	1.31	Max	1.41	CHE	1.24		
EKS	**1.30**	**Fish**	**1.27**	**ExR**	**1.17**		

TABLE 4.— OECD DATA 1993 - AVERAGE PRICE AND AVERAGE BASKET METHODS (CAN/US PPPs)

Average Price PPPs				Average Basket PPPs			
DEU	1.26	CHE	1.27	DEU	1.23	CHE	1.22
FRA	1.30	FIN	1.34	FRA	1.23	FIN	1.24
ITA	1.33	ICE	1.30	ITA	1.28	ICE	1.25
NLD	1.27	NOR	1.32	NLD	1.20	NOR	1.20
BEL	1.29	SWE	1.30	BEL	1.24	SWE	1.27
LUX	1.27	TUR	1.33	LUX	1.28	TUR	1.26
GBR	1.30	AUS	1.31	GBR	1.29	AUS	1.22
IRL	1.31	NZL	1.30	IRL	1.28	NZL	1.26
DNK	1.29	JPN	1.27	DNK	1.25	JPN	1.19
GRC	1.34	CAN	1.29	GRC	1.29	CAN	1.21
ESP	1.32	USA	1.21	ESP	1.23	USA	1.29
PRT	1.37	Min	1.21	PRT	1.23	Min	1.19
AUT	1.28	Max	1.37	AUT	1.24	Max	1.29
EKS	**1.27**	**Fish**	**1.25**	**ExR**	**1.29**		