What Impact Does the Choice of Formula have on International Comparisons?

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February 9, 1998

Abstract

This paper undertakes an empirical comparison of a number of alternative multilateral index-number formulae. It ascertains the magnitude of the effect of choosing one formula over another using an appropriate cross-sectional data set constructed under the auspices of the the Eurostat-OECD Purchasing Power Parity Programme. In aid of this, a new indicator is proposed which facilitates the measurement of the difference between two sets of bloc consumption shares, each computed using a different multilateral comparison method.

Key Words: Index numbers; multilateral comparisons; purchasing power parities.

JEL Classification Numbers: C43, C81, E31, F31, O57.

1 Introduction

Many different methods for aggregating microeconomic price and quantity data into a multilateral index of real output (or a component thereof like consumption) have been proposed during the past ninety years. All along, some of these methods have been put into practice by various statistical agencies and international organizations for the purposes of economic analysis and/or public policy. More recently, attempts have been made to find a theoretical justification for the choice of one method over all others. The most fruitful path taken thus far in attempting to provide such a justification is arguably the test (or axiomatic) approach. Using this approach, the investigator specifies a set of "reasonable" tests and then uses it as the basis for assessing the relative merits of alternative independently-motivated methods.[1] Diewert (1986) proposed what has come to be viewed as the benchmark set of multilateral tests and used it to assert the superiority of three methods[2] in relation to five others. Using a modified version of Diewert's set, Balk (1996) asserted the superiority of two different methods[3] in relation to eight others (including two of the three methods Diewert deemed

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[0] The author wishes to thank Erwin Diewert, Ken White, Chuck Blackorby, John Helliwell and two anonymous referees for helpful comments on earlier versions of this paper.

[1] The ultimate objective of showing that a set of reasonable tests completely characterizes a particular multilateral comparison formula has never been realized.

[2] *Viz.*, the "star," EKS and own-share methods.

[3] *Viz.*, the van Ijzeren balanced and Geary-Khamis methods.

superior). Setting aside the debate over which of these methods are "best," there is a clear consensus of opinion among experts in the field of international comparisons that almost any method is better than simply converting economic aggregates into a numéraire currency by means of exchange rates. If almost any method is better than the exchange-rate approach, does the choice among these superior methods matter very much? The present paper aims to show is that it does: that the choice of one method (or formula) over another can have a substantial impact on the resulting international comparisons.

The question of how to compare multilateral real-output or purchasing-power-parity formulae from an empirical standpoint has received scant attention in the literature. If the formulae under consideration satisfy a certain minimal set of requirements, then the application of any one of them to a bloc consisting of K countries yields a vector of $K - 1$ numbers which can serve as a basis for all possible binary comparisons within the bloc. The universal means by which two such vectors have been compared in the past has been an assessment of the component-wise percentage differences between them.[4] This approach is unsatisfactory for a couple of reasons. First, the percentage difference between two numbers is an asymmetric indicator of the relative difference between them because it depends on which number is used as the point of comparison. To paraphrase an example from Törnqvist

[4] See, for example, Kravis *et al.* (1975, chs. 1 and 5) and Ruggles (1967). Note that "similarity indexes" such as those calculated by Kravis, Heston and Summers (1982, ch. 9) measure the similarity between two vectors of prices or quantities with reference to a *single* multilateral comparison formula.

et al. (1985, p. 43), 250 is twenty-five percent more than 200, or 200 is twenty percent less than 250. Second, component-wise comparisons between two vectors are unlikely to give rise to a very accurate assessment of the overall difference between them unless the components are few in number or the calculated differences exhibit little variation in size.

Section 2 proposes a new index of the difference between the results of two multilateral comparison methods applied to the same data set. Based on the normed, symmetric and additive log(arithmic) difference indicator, this index overcomes the problems mentioned above to provide an appropriate summary measure of the differences between the purchasing power parities (PPPs) or output shares associated with the two methods. Section 4 describes the data used in section 5 to undertake an empirical comparison of the twelve specific methods—two new ones and ten proposed elsewhere—described in section 3. Section 6 concludes by explaining why different sources provide different values for the same PPPs.

2 A Summary Measure of the Differences between Alternative Formulae

The maintained domain of comparison consists of a bloc of countries $\mathcal{K} := \{1, \ldots, K\}$ with $H := (H_1, \ldots, H_K)' \in \mathbb{R}^K_{++}$ resident households, a set of consumer goods and services $\mathcal{N} := \{1, \ldots, N\}$ with country-specific national-currency-denominated prices

$$P := (p^1, \ldots, p^K) = \begin{pmatrix} p^1_1 & \cdots & p^K_1 \\ \vdots & & \vdots \\ p^1_N & \cdots & p^K_N \end{pmatrix} \in \mathbb{R}^{NK}_{++} ,$$

and a vector of per-household consumption bundles

$$X := (x^1, \ldots, x^K) = \begin{pmatrix} x_1^1 & \ldots & x_1^K \\ \vdots & & \vdots \\ x_N^1 & \ldots & x_N^K \end{pmatrix} \in \mathbb{R}_+^{NK} \ .$$

Thus each of the H_k households in country $k \in \mathcal{K}$ is considered to be the purchaser of $x_n^k \geq 0$ units of commodity $n \in \mathcal{N}$ at a price of $p_n^k > 0$ country-k currency units. Following the conventions of the test approach, the underlying preferences which generate X are ignored and the elements of P, X and H are treated as independent variables.

An axiomatic PPP index for country i relative to country j is a function $\rho^{ji} : \mathbb{R}_{++}^{KN} \times \mathbb{R}_+^{K(N+1)} \to \mathbb{R}$ with image $\rho^{ji}(P, X, H)$. It is assumed that, at the very least, this index is positive and transitive with respect to j and i. The *positivity* requirement enables the usual interpretation of $\rho^{ji}(P, X, H)$ as the number of country-i currency units needed to buy a commodity bundle equivalent to one that can be bought with a single country-j currency unit.

P. *Positivity*: For all $(j, i) \in \mathcal{K} \times \mathcal{K}$,

$$\rho^{ji}(P, X, H) > 0 \ .$$

The *transitivity* requirement guarantees that the results of applying ρ^{ji} to a bloc comprising three or more countries are self-consistent.

T. *Transitivity*: For all $(j, i) \in \mathcal{K} \times \mathcal{K}$ and for all $l \in \mathcal{K}$,

$$\rho^{jl}(P, X, H)\rho^{li}(P, X, H) = \rho^{ji}(P, \ X, H) \ .$$

In addition to satisfying T, a self-consistent set of PPPs has two further properties. The first, called *weak identity*, requires the value of ρ^{ji} to be unity when $i = j$.

WI. *Weak Identity*: For all $j \in \mathcal{K}$,

$$\rho^{jj}(P, X, H) = 1 \ .$$

The second, *country reversal*, asserts that the value of ρ^{ij} is the reciprocal of the value of ρ^{ji}.

CR. *Country Reversal*: For all $(j, i) \in \mathcal{K} \times \mathcal{K}$,

$$\rho^{ij}(P, X, H) = \frac{1}{\rho^{ji}(P, X, H)} \ .$$

Theorem 1 *If ρ^{ji} satisfies P and T then it also satisfies WI and CR.*

The proof of this and all subsequent theorems can be found in the appendix.

Consider two sets of PPPs, A and B, each computed using a different multilateral index-number formula satisfying P and T. For ease of exposition, let the K-dimensional square matrices $\left(\rho_A^{ji}\right)$ and $\left(\rho_B^{ji}\right)$ represent the elements of A and B, respectively. Since P together with T implies WI so that $\rho_A^{jj} = \rho_B^{jj} = 1$ for all $j \in \mathcal{K}$, there are up to $K^2 - K$ possible differences between these matrices. One way to construct a summary measure of these differences is to calculate their mean. For such an index to be meaningful, however, the elementary difference indicator must be unit-independent; *i.e.*, it must measure the *relative* difference between ρ_A^{ji} and ρ_B^{ji}.[5]

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[5] The *absolute* difference $\rho_B^{ji} - \rho_A^{ji}$ is of the dimensionality $i\$/j\$$—the number of units of country

Vartia (1974, p. 5) defined an indicator of relative difference as a function $d : \mathbb{R}_+^2 \to \mathbb{R}$ which is continuous, increasing, homogeneous of degree zero and has the additional property that

$$d\left(\rho_A^{ji}, \rho_B^{ji}\right) \underset{(=)}{\geq} 0 \text{ if and only if } \rho_B^{ji} \underset{(=)}{\geq} \rho_A^{ji} \ .$$

The homogeneity property implies that only the ratio ρ_B^{ji}/ρ_A^{ji} matters; *i.e.*, there exists a function $D : \mathbb{R}_+ \to \mathbb{R}$ such that $D\left(\rho_B^{ji}/\rho_A^{ji}\right) \equiv d\left(1, \rho_B^{ji}/\rho_A^{ji}\right)$. Obviously, D is continuous, increasing, homogeneous of degree zero and satisfies

$$D\left(\rho_B^{ji}/\rho_A^{ji}\right) \underset{(=)}{\geq} 0 \text{ if and only if } \rho_B^{ji}/\rho_A^{ji} \underset{(=)}{\geq} 1 \ .$$

Examples of this function can be found in Törnqvist *et al.* (1985, p. 44).

As explained in section 1, the problem with using percentage difference as an indicator of relative difference is that it is asymmetric. Formally, the indicator D is said to be symmetric if

$$D\left(\rho_A^{ji}/\rho_B^{ji}\right) = -D\left(\rho_B^{ji}/\rho_A^{ji}\right) \ .$$

It is easy to show that the percentage difference indicator $D_1\left(\rho_B^{ji}/\rho_A^{ji}\right) := \rho_B^{ji}/\rho_A^{ji} - 1$ does not satisfy this requirement.

Since the index-number formulae generating $\left(\rho_A^{ji}\right)$ and $\left(\rho_B^{ji}\right)$ are assumed to be transitive, it would be desirable if the relative difference with respect to any two countries were equal to the relative difference with respect to the first country and some third country plus

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i's currency per unit of country j's.

the relative difference with respect to the same third country and the second country; *i.e.*, for any $l \in \mathcal{K}$,

$$D\left(\frac{\rho_B^{ji}}{\rho_A^{ji}}\right) = D\left(\frac{\rho_B^{jl}}{\rho_A^{jl}}\right) + D\left(\frac{\rho_B^{li}}{\rho_A^{li}}\right) \ . \tag{1}$$

This property is equivalent to D being additive:

$$D\left(\frac{\rho_B^{jl}}{\rho_A^{jl}}\frac{\rho_B^{li}}{\rho_A^{li}}\right) = D\left(\frac{\rho_B^{jl}}{\rho_A^{jl}}\right) + D\left(\frac{\rho_B^{li}}{\rho_A^{li}}\right) \ . \tag{2}$$

Using T and then CR, equation (1) can be re-written as

$$D\left(\frac{\rho_B^{ji}}{\rho_A^{ji}}\right) = D\left(\frac{\rho_B^{ji}}{\rho_A^{ji}}\frac{\rho_A^{li}}{\rho_B^{li}}\right) + D\left(\frac{\rho_B^{li}}{\rho_A^{li}}\right) \ .$$

Setting $j = i$ and then invoking WI shows that an additive indicator of relative difference is symmetric. Letting $u := \rho_B^{jl}/\rho_A^{jl}$ and $v := \rho_B^{li}/\rho_A^{li}$, equation (2) can be re-written as

$$D\left(uv\right) = D\left(u\right) + D\left(v\right) \ .$$

Eichhorn (1978, p. 13) showed that the only solution to this functional Cauchy-type equation is $D\left(u\right) = \alpha \ln u$, $\alpha \in \mathbb{R}$.

Another desirable property for the indicator D would be that it behave approximately as D_1 when $\rho_B^{ji}/\rho_A^{ji} \approx 1$. Formally, D is said to be *normed* if

$$\lim_{u \to 1} \frac{D\left(u\right)}{D_1\left(u\right)} = 1$$

$$\Leftrightarrow \lim_{u \to 1} \frac{D\left(u\right) - D\left(1\right)}{u - 1} =: D'\left(u\right) = 1 \ .$$

Thus, requiring D to be normed as well as symmetric and additive means that it must be the log difference indicator $\ln\left(\rho_B^{ji}/\rho_A^{ji}\right)$.

The symmetry property of this indicator in conjunction with CR implies that the mean of the $K^2 - K$ log differences between $\left(\rho_A^{ji}\right)$ and $\left(\rho_B^{ji}\right)$ is zero. Consequently, the appropriate summary measure of these differences is the sum of the *absolute* log differences between corresponding off-diagonal elements divided by their number:

$$\Delta_{A,B} = \frac{\sum_{j=1}^{K} \sum_{i \neq j} \left| \ln \left(\frac{\rho_B^{ji}}{\rho_A^{ji}} \right) \right|}{K(K-1)} \tag{3}$$

Using T, for any $l \in \mathcal{K}$, (3) can be re-written as

$$\begin{aligned} \Delta_{A,B} &= \frac{\sum_{j=1}^{K} \sum_{i \neq j} \left| \ln \left(\frac{\rho_B^{jl} \rho_B^{li}}{\rho_A^{jl} \rho_A^{li}} \right) \right|}{K(K-1)} \\ &= \frac{2}{K(K-1)} \sum_{j=1}^{K-1} \sum_{i=j+1}^{K} \left| \ln \left(\frac{\rho_B^{li}}{\rho_A^{li}} \right) - \ln \left(\frac{\rho_B^{lj}}{\rho_A^{lj}} \right) \right| \text{, by CR.} \end{aligned} \tag{4}$$

For a particular bloc of countries, let $\mathcal{P} := \{A, B, \ldots\}$ denote the set of all sets of PPPs which satisfy P and T.

Theorem 2 *Δ defined by the right-hand side of (4) is a metric on $\mathcal{P}$; i.e., $\Delta_{\cdot,\cdot}$ is a real-valued function on $\mathcal{P} \times \mathcal{P}$ which satisfies (i) $\Delta_{A,B} \geq 0$; (ii) $\Delta_{A,B} = 0$ if and only if $A = B$; (iii) $\Delta_{A,B} = \Delta_{B,A}$; and (iv) $\Delta_{A,B} \leq \Delta_{A,C} + \Delta_{C,B}$ (triangle inequality).*

Thus Δ possesses the most important properties of ordinary distance in $\mathbb{R}^3$ making it a reasonable and intuitive measure of the difference between alternative sets of PPPs.

Table 1 contains three sets of PPPs covering the same twenty-four countries: the first two were calculated by the Organization for Economic Cooperation and Development (OECD) using the Eltetö-Köves-Szulc (EKS) method and the Geary-Khamis (GK) method,[6]

[6] These methods are described in the next section.

respectively; the third was calculated for the Penn World Table (PWT) using the GK method. For comparison, the corresponding exchange rates (ER) are also included in table 1. The differences among these four sets of numbers can be summarized by computing the associated Δ values using equation (4): $\Delta_{EKS,GK} = 0.04773$, $\Delta_{EKS,PWT} = 0.10425$, $\Delta_{GK,PWT} = 0.08354$, $\Delta_{EKS,ER} = 0.28832$, $\Delta_{GK,ER} = 0.30469$ and $\Delta_{PWT,ER} = 0.34659$. Thus the OECD PPPs differ from one another by about 4.8 percent and from the exchange rates by roughly thirty percent, and the GK PPPs differ from one another by about 8.4 percent[7] and from the exchange rates by over thirty percent.

A system of bloc-specific (real) consumption indexes for countries $1, \ldots, K$ is a function $\sigma : \mathbb{R}^{KN}_{++} \times \mathbb{R}^{K(N+1)}_{+} \rightarrow \mathbb{R}$ with image $\sigma(P, X, H) := [\sigma^1(P, X, H), \ldots, \sigma^K(P, X, H)]'$. To enable the i^{th} element ($i \in \mathcal{K}$) of this system to be interpreted as country i's share of total bloc consumption, σ is required to satisfy

S1. *Fundamental Share Test*: $\sigma^i(P, X, H) > 0$ for all $i \in \mathcal{K}$ and $\sum_{i=1}^{K} \sigma^i(P, X, H) = 1$.

Theorem 3 *If σ satisfies S1 then ρ^{ji} defined implicitly by*

$$\rho^{ji}(P, X, H)\frac{\sigma^i(P, X, H)}{\sigma^j(P, X, H)} = \frac{H_i \, p^{i\prime} x^i}{H_j \, p^{j\prime} x^j} \tag{5}$$

satisfies P and T, and

$$\sigma^i(P, X, H) = \left\{ \sum_{j=1}^{K} \frac{H_j \, p^{j\prime} x^j}{H_i \, p^{i\prime} x^i} \rho^{ji}(P, X, H) \right\}^{-1} . \tag{6}$$

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[7] The reason for this difference is provided at the end of section 5.

Under the assumptions of this theorem, the number $\rho^{ji}(P, X, H)$ is the amount by which the total bloc expenditure of country-i households relative to those of country j must be deflated in order to make it equal to the corresponding total consumption ratio.

Substituting for ρ^{lk} in (4) using (5) yields an equivalent expression for the mean absolute log difference between multilateral comparison methods A and B:

$$\Delta_{A,B} = \frac{2}{K(K-1)} \sum_{j=1}^{K-1} \sum_{i=j+1}^{K} \left| \ln\left(\frac{\sigma_B^i}{\sigma_A^i}\right) - \ln\left(\frac{\sigma_B^j}{\sigma_A^j}\right) \right| . \tag{7}$$

Thus $\Delta_{A,B}$ can be calculated from associated basis sets of PPPs using (4) or from the associated consumption-share systems using (7).

3 Some Specific Methods

As noted above, Diewert (1986) evaluated eight specific multilateral comparison formulae in the light of his test approach. The empirical work of section 5 compares results from seven of these formulae, two variants of the eighth and three others, two of which are new. The present section provides a brief description of each.

The two new formulae proposed here are motivated by the notion that each binary national- price-level comparison should be an average of the corresponding comparisons made from the perspectives of the K "average households." Given the available data, household k's best price-level comparison between country i and country j is obtained by taking the ratio of the results of pricing x^k at both p^i and p^j. This index will be approximately exact if the household has preferences which admit very little substitution among the N commodities, or if the K price vectors are not very different from one another.

Let $\theta^k(H) := H_k / \sum_{l=1}^K H_l$ denote the fraction of bloc households living in country k. The household-share-weighted geometric mean of the K average-household PPP indexes is called the *household democratic PPP index for country i relative to country j*:

$$\rho_{HD}^{ji}(P, X, H) := \prod_{k=1}^{K} \left[\frac{p^{i\prime} x^k}{p^{j\prime} x^k} \right]^{\theta^k(H)} . \tag{8}$$

By assigning each average-household index a weight which is proportional to the corresponding total number of households, ρ_{HD}^{ji} affords equal treatment to all households in the bloc.

A weaker democratic aggregation rule would treat countries as equals rather than households. Accordingly, the *country democratic PPP index for country i relative to country j* is defined as the unweighted geometric mean of the K average-household PPP indexes:

$$\rho_{CD}^{ji}(P, X, H) := \prod_{k=1}^{K} \left[\frac{p^{i\prime} x^k}{p^{j\prime} x^k} \right]^{\frac{1}{K}} . \tag{9}$$

The converse of averaging over the relative costs of consumption bundles X using prices p^i and p^j is to compute the relative cost of the bloc average consumption bundle $\chi(X, H) := \sum_{k=1}^K \theta^k(H) x^k$ using p^i and p^j. This method, first used by the United Nations Economic Commission for Latin America (ECLA) in the 1960s, is called the *ECLA* or *average basket PPP index for country i relative to country j*:

$$\rho_{AB}^{ji}(P, X, H) := \frac{p^{i\prime} \chi(X, H)}{p^{j\prime} \chi(X, H)} . \tag{10}$$

Early multilateral comparison methods were based on bilateral index-number formulae. The simplest and most popular of these methods involved the use of the Laspeyres formula in making binary quantity comparisons between a pre-selected base country and each of the other countries in the bloc. In general, such a "star system" can be constructed using any index-number formula of the form $\phi(p^j, p^i, x^j, x^i)$. Accordingly, for a given base country $k \in \mathcal{K}$, the *country-k star system of consumption shares* is defined by

$$\sigma^i_{k*}(P, X, H) := \frac{H_i\, \phi(p^k, p^i, x^k, x^i)}{\sum_{j=1}^{K} H_j\, \phi(p^k, p^j, x^k, x^j)}\ . \tag{11}$$

A second multilateral comparison method based on a bilateral formula is due to Gini (1931, p. 12). Known by the initials of its three independent re-discoverers, Eltetö and Köves (1964) and Szulc (1964), the (*generalized*) *EKS system of consumption shares* is defined by[8]

$$\sigma^i_{EKS}(P, X, H) := \frac{H_i\, \prod_{k=1}^{K} \left[\phi(p^k, p^i, x^k, x^i)\right]^{\frac{1}{K}}}{\sum_{j=1}^{K} H_j\, \prod_{l=1}^{K} \left[\phi(p^l, p^j, x^l, x^j)\right]^{\frac{1}{K}}}\ . \tag{12}$$

A third bilateral-formula-based multilateral comparison method is due to Diewert (1986, p. 25). His *own-share system of consumption indexes* is defined by

$$\sigma^i_{OS}(P, X, H) := \frac{H_i \left\{\sum_{k=1}^{K} H_k \left[\phi(p^k, p^i, x^k, x^i)\right]^{-1}\right\}^{-1}}{\sum_{j=1}^{K} H_j \left\{\sum_{l=1}^{K} H_l \left[\phi(p^l, p^j, x^l, x^j)\right]^{-1}\right\}^{-1}}\ . \tag{13}$$

Note that in the own-share and EKS systems, respectively, a harmonic mean of bilateral comparisons and a geometric mean of bilateral comparisons substitute for the base-country bilateral comparison of the star system.

The next three multilateral methods are based on weighted averages of the country-k star systems. Respectively, the *democratic weights*, *plutocratic weights* and *quantity weights*

[8] In the version of this index advanced by Eltetö and Köves (1964) and Szulc (1964), the Fisher formula was used in place of ϕ.

consumption-share systems are defined by

$$\sigma^i_{DW}(P,X,H) := \sum_{k=1}^{K} \tfrac{1}{K}\, \sigma^i_{k*}(P,X,H) \ , \tag{14}$$

$$\sigma^i_{PW}(P\hat{r},X,H) := \sum_{k=1}^{K} s^k(P\hat{r},X,H)\sigma^i_{k*}(P\hat{r},X,H) \tag{15}$$

and

$$\sigma^i_{QW}(P,X,H) := \sum_{k=1}^{K} \sigma^k_{OS}(P,X,H)\sigma^i_{k*}(P,X,H) \ , \tag{16}$$

where

$$s^k(P\hat{r},X,H) := \frac{H_k \left(r_k p^k\right)' x^k}{\sum_{l=1}^{K} H_l \left(r_l p^l\right)' x^l} \tag{17}$$

is country k's share of (nominal) bloc expenditure, $r := (r_1, \ldots, r_K)'$ is a vector of exchange rates and $\hat{r}$ is the $K \times K$ diagonal matrix with $\hat{r}_{kk} = r_k$ for all $k \in \mathcal{K}$.[9]

None of the three remaining multilateral methods are based on a bilateral formula. The first is a proposal by Geary (1958) which was later amplified by Khamis (1970)(1972); the second and third are variants of van Ijzeren's (1956) weighted balanced method.

The Geary-Khamis or GK consumption shares are found by solving the following system of equations:

$$\begin{aligned} \sigma_i &= \sum_{n=1}^{N} \pi_n \left[H_i\, x^i_n\right] \ , \quad i = 1, \ldots, K, \\ \pi_n &= \frac{\sum_{i=1}^{K} \omega^i_n \sigma_i}{\sum_{k=1}^{K} H_k\, x^k_n} \ , \quad n = 1, \ldots, N, \end{aligned} \tag{18}$$

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[9] Since r_k is the price of a unit of country k's currency in terms of some numéraire currency, $P\hat{r}$ is the matrix of numéraire-denominated bloc commodity prices.

where $\omega_n^i := p_n^i x_n^i / p^{i\prime} x^i$ is the n^{th} country-i per-household expenditure share. Equations (18b) define the "international price" of each commodity as the ratio of the per-household expenditure-share-weighted sum of the K consumption shares to the total quantity consumed. Equations (18a) define the share of bloc consumption for each country as the cost of its national basket at international prices.

The $N + K$ equations (18) are not independent since each constituent set implies

$$\sum_{n=1}^{N} \pi_n \sum_{i=1}^{K} H_i \, x_n^i = \sum_{i=1}^{K} \sigma_i \tag{19}$$

and, consequently, at least one non-trivial solution exists. Khamis (1970, section 3) showed that, subject to any normalization on the σ_i s,[10] the system consisting of any $N + K - 1$ of the equations (18) has a unique positive solution.

The consumption shares associated with van Ijzeren's weighted balanced method are found by solving the following system of equations:

$$\sum_{k \neq i} \alpha_k \, \frac{p^{i\prime} x^k}{p^{i\prime} x^i} \frac{H_k}{H_i} \frac{\sigma_i}{\sigma_k} = \sum_{k \neq i} \alpha_k \, \frac{p^{k\prime} x^i}{p^{k\prime} x^k} \frac{H_i}{H_k} \frac{\sigma_k}{\sigma_i} \ , \ i = 1, \ldots, K, \tag{20}$$

where α_k is the country-k "weighting coefficient." If $\xi_1 \equiv p^{1\prime}(H_1 \, x^1)/\sigma_1, \ldots, \xi_K \equiv p^{K\prime}(H_K \cdot x^K)/\sigma_K$ are called "equivalents," the left-hand side of (20) is the number of equivalents that would be required to buy, in country i, the quantities in the weighted national baskets that can be bought for one equivalent in countries $1, \ldots, i-1, i+1, \ldots, K$. The right-hand side is the number of equivalents that would be required to buy, in each of countries

[10] *E.g.*, $\sum_{i=1}^{K} \sigma_i = 1$.

$1, \ldots, i-1, i+1, \ldots, K$, the weighted quantities purchased in country i for one equivalent. The balanced method asserts that, for $i = 1, \ldots, K$, these two quantities of money are equal.

Van Ijzeren (1956, pp. 25–27) showed that, subject to any normalization on the σ_i s, the system consisting of any $K-1$ of equations (20) has a unique positive solution. Under the normalization $\sum_{i=1}^{K} \sigma_i = 1$, this system is referred to as the household-weighted balanced (VH) method if $\alpha_k := H_k$ and the quantity-weighted balanced (VQ) method if $\alpha_k := \sigma_k$. The former weighting scheme originates with van Ijzeren (1956, p. 4); the latter with van Ijzeren (1983, p. 45).

4 The Data

The raw price and expenditure data used in the empirical work of the next section are those of the Eurostat-OECD PPP Programme. These data cover the bloc comprising the twenty-four OECD countries of 1990 and the general commodity list made up of the 158 basic headings[11] of the major aggregate called "Final Consumption of Resident Households." Let $V := (v_n^k)$ denote the (158×24) matrix of national expenditures (in national currency units) at the basic heading level, and let $\bar{P} := (\bar{p}_n^k)$ denote the corresponding matrix of basic-heading

[11] In principle, a basic heading consists of a small group of similar well-defined goods or services. In practice, it is the lowest level of classification for which expenditures can be estimated. Consequently, an actual basic heading can cover a broader range of commodities than is theoretically desirable.

PPPs in national currency units per U.S. dollar. Hence, for all $n \in \mathcal{N}$ and for all $k \in \mathcal{K}$,

$$v_n^k \equiv H_k \, p_n^k x_n^k \tag{21}$$

and

$$\bar{p}_n^k \equiv \frac{p_n^k}{p_n^{US}} \ . \tag{22}$$

Several different sources were employed in the determination of the household numbers (H) presented in table 2. For the United States and Japan, Turkey, each of the Nordic countries excluding Iceland,[12] and each of the European Union countries excluding Denmark and Germany, the corresponding datum was furnished by, respectively, the United Nations (1993, table 3), the State Institute of Statistics (1993, table 65), the Nordic Statistical Secretariat (1994, table 105), and Eurostat (1992, table 3.13). Estimates of H_k for Austria, Switzerland, Canada, Australia and New Zealand were made by linear interpolation using, respectively, the corresponding 1983 and 1993 figures reported by Eurostat (1992, table 3.13) (1995, table 3.13), the 1980 figure reported by the United Nations (1993, table 3) and the 1993 figure reported by Eurostat (1995, table 3.13), the 1986 figure reported by the United Nations (1993, table 3) and the 1991 figure reported by Statistics Canada (1992, table 8), the 1986 and 1991 figures reported by the Australian Bureau of Statistics (1995, table 5.11), and the 1986 and 1991 figures reported by the United Nations (1993, table 3). Linear extrapolation was used to estimate H_k for Germany[13] based on the corresponding

[12] *Viz.*, Denmark, Finland, Norway and Sweden.

[13] More precisely, the Federal Republic of Germany, including West Berlin, as constituted prior

1986 and 1989 figures reported by Eurostat (1989, table 3.13) (1992, table 3.13). For Iceland, which has not undertaken a national census in over thirty-five years, it was assumed that the population-household ratio in 1990 was the same in relation to the range of values exhibited by Norway, Sweden and Finland as it was in 1960.[14] The number of households was computed by using this estimated ratio to deflate the corresponding population value.

For any scale factor $\beta \in \mathbb{R}_{++}$, a matrix $\bar{X} := (\bar{x}_n^k)$ of scaled per-household quantities consistent with $(V, \bar{P}, H)$ is defined as

$$\bar{x}_n^k := \frac{\beta v_n^k}{\bar{p}_n^k H_k} \tag{23}$$

$$\equiv \beta p_n^{US} x_n^k \text{ , by (21) and (22).} \tag{24}$$

In the present paper, a β-value of 100,000 was used in the construction of $\bar{X}$ so that its elements could be treated as integers without any significant loss of precision.

5 Empirical Results

The calculation of PPPs based on the data set $(\bar{P}, \bar{X}, H)$ can only be accomplished by means of formulae which, in addition to satisfying P and T, satisfy two other axioms. The first of these, called *quantity dimensionality with respect to* X, requires that a common proportional change in all per-household quantities have no effect on the value of ρ^{ji}.

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to October 3, 1990.

[14] The 1960 population and household numbers for Iceland, Norway, Sweden and Finland were furnished by the OECD (1993a, p. 156) and the United Nations (1993, table 3).

QDX. *Quantity Dimensionality with Respect to X:* For all $(j,i) \in \mathcal{K} \times \mathcal{K}$ and for all $\beta \in \mathbb{R}_{++}$,

$$\rho^{ji}(P, \beta X, H) = \rho^{ji}(P, X, H) \ .$$

The second, *commensurability*, requires that a change in the unit of measure of each commodity have no effect on the value of ρ^{ji}.

C. *Commensurability*: For all $(j,i) \in \mathcal{K} \times \mathcal{K}$ and for all $\lambda := (\lambda_1, \ldots, \lambda_K)' \in \mathbb{R}^N_{++}$,

$$\rho^{ji}(\hat{\lambda} P, \hat{\lambda}^{-1} X, H) = \rho^{ji}(P, X, H) \ ,$$

where $\hat{\lambda}$ is the $N \times N$ diagonal matrix with $\hat{\lambda}_{nn} = \lambda_n$ for all $n \in \mathcal{N}$.

Theorem 4 *$\rho^{ji}(\bar{P}, \bar{X},\ H) = \rho^{ji}(P, X, H)$ if and only if ρ^{ji} satisfies QDX and C.*

Similarly, the calculation of consumption shares based on the data set $(\bar{P}, \bar{X}, H)$ can be facilitated only by formulae which satisfy S1 and two other axioms. The first of these, called the *monetary unit test with respect to X*, states that multiplying the matrix of per-household quantities by a positive scalar has no effect on the consumption share of any country.

S4X. *Monetary Unit Test with Respect to X:* For all $i \in \mathcal{K}$ and for all $\beta \in \mathbb{R}_{++}$,

$$\sigma^{i}(P, \beta X, H) = \sigma^{i}(P, X, H) \ .$$

The second, *share commensurability*, requires the consumption shares to be invariant to changes in the units of measure of commodities.

S5. *Share Commensurability*: For all $i \in \mathcal{K}$ and for all $\lambda := (\lambda_1, \ldots, \lambda_K)' \in \mathbb{R}^N_{++}$,

$$\sigma^i(\hat{\lambda}P, \hat{\lambda}^{-1}X, H) = \sigma^i(P, X, H) \ .$$

Theorem 5 *$\sigma(\bar{P}, \bar{X}, H) = \sigma(P, X, H)$ if and only if σ satisfies S4X and S5.*

All twelve of the multilateral comparison methods presented in section 3 satisfy S1, S4X and S5 or, equivalently, by theorem 3, P, T, QDX and C. Consequently, the computation of consumption shares was a straightforward exercise involving simple substitutions into the defining formulae. Table 3 contains a selection of the results of this exercise. Included are the two new methods—HD and CD—three of the four methods not based on a bilateral formula—AB, GK and VH—and three of the six bilateral-formula–based methods—EKS, OS and the k-star with $k := US$ (US*). The GK and VH consumption-share systems were each calculated iteratively using the household-democratic consumption shares as initial values. The Fisher "ideal" consumption index ϕ_F defined by

$$\phi_F(p^j, p^i, x^j, x^i) := \left[\frac{p^{j\,\prime} x^i}{p^{j\,\prime} x^j} \frac{p^{i\,\prime} x^i}{p^{i\,\prime} x^j} \right]^{\frac{1}{2}} \tag{25}$$

was used as the basis for each of the bilateral-formula -based methods.

The mean absolute log differences among the eight methods of table 3 and the exchange-rate approach[15] are expressed as percentages in table 4. If the cutoff between

[15] The exchange-rate–based consumption-share system was calculated by substituting the country-i exchange rate with respect to country j, as reported in OECD (1992, table 2.5), for $\rho^{ji}(P, X, H)$ in equation (6).

"substantial" and "insubstantial" is set at two percent, this table partitions the considered methods into five groups based on whether or not they are substantially different from one another.[16] HD, AB and CD are grouped together since all of the differences among them lie below the cutoff while all of the differences involving just one of them lie above. Similarly, VH, EKS and OS[17] form a group[18] as do each of GK, US* and ER.[19] Thus the choice of one method over another can have a substantial impact on international comparisons of consumption.

Table 5 presents eight per-household consumption indexes derived from the results in table 3 using the household numbers in table 2. Each of these indexes measures the consumption of the average household in each OECD country as a percentage of that in the United States. Figure 1 is a graphical representation of selected results in table 5 along

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[16] To get a feel for what this means, consider a hypothetical international project to be financed by reference to 1990 OECD consumption shares. Using the own-share system instead of the US* system ($100\Delta_{US^*,OS} \approx 2$) would change the average national contribution by 1.58 percent. For some countries, however, this switch in methods would change their contribution by as much as 3.99 percent. In an era of government fiscal restraint, four dollars per hundred can easily be viewed as a substantial difference.

[17] Since it is so close to two, $100\Delta_{US^*,OS} = 1.992$ is treated as a substantial difference.

[18] An extended version of table 4 would show that the quantity-weighted van Ijzeren, democratic weights, plutocratic weights and quantity weights methods also belong to this group.

[19] Preliminary support for the hypothesis that this partition is not merely an artifact of the (1990) data is given by the fact that the same partition obtained from the corresponding 1993 data.

with those of the exchange-rate approach. Therein, the relevant countries[20] are arranged from left to right along the horizontal axis in order of decreasing per-household consumption calculated via the household democratic method.

Constructed in the same manner as figure 1, the next three figures serve to illustrate the difference partition established above. The close proximity of the per-household consumption lines in each of figures 2 and 3 conveys the similarity of outcomes generated by methods belonging to the same group. By contrast, the relative separation of the corresponding lines in figure 4 conveys the dissimilarity between methods belonging to different groups.

In table 6, each entry is a PPP associated with the corresponding consumption share in table 3 by means of equation (5). Comparison of tables 6 and 1 shows that there are relatively large differences between the PWT(-GK) PPPs and the GK PPPs calculated by both the author and the OECD. For instance, the PPP for Canada is 1.31 according to the former and 1.16 according to the latter—a difference of about twelve percent. Given that both sets of numbers were calculated using the same formula, why do they differ so much? The answer to this question was provided by Hill (1982, p. 7):

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[20] The United States (US), Luxembourg (LUX), Canada (CAN), Switzerland (CHE), Japan (JAP), Australia (AUS), Italy (ITA), France (FRA), Belgium (BEL), Germany (GER), the United Kingdom (UK), New Zealand (NZ), Iceland (ICE), Austria (AUT), Spain (SPA), the Netherlands (NLD), Ireland (IRE), Finland (FIN), Denmark (DEN), Sweden (SWE), Norway (NOR), Greece (GRC), Portugal (PRT) and Turkey (TUR).

> It is inherent in all multilateral aggregation methods ... that the characteristics of the group of countries as a whole impose themselves on measurements made within the group. [The PPP] of France [relative to] Germany, for example, cannot be exactly the same within the context of the [European Union] as it appears within the context of the world economy as a whole, even if exactly the same basic[-heading] PPPs, [national] expenditures and aggregation method are used.

With respect to the GK PPPs of the present paper, then, the noted differences are due to the fact that one set was calculated in the context of the 152-country PWT whereas the other two were calculated in the context of the 24-country OECD.[21] Consequently, it seems sensible to regard the OECD-GK PPPs and the corresponding PWT PPPs as distinct concepts rather than as different measures of the same concept.

Comparison of tables 6 and 1 also reveals small differences between the (non- PWT-) GK and EKS PPPs.[22] For both methods, these differences have arisen because the definition of private final consumption expenditure employed herein excludes expenditures by private non-profit institutions serving households whereas that of the OECD does not. For the EKS method, the differences are also due to the imposition of "fixity" by the OECD. Under this requirement, the "official" PPPs for the European Union (EU) must remain unchanged in any comparison involving a larger group of countries. The achievement of fixity is a two-step

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[21] The "consistentization" procedure used in the construction of the PWT (PWT Appendix, Dec. 1994, endnote /4/) had no effect on the component-level PPPs used in the present paper. Under this procedure, the "adjustment factor" for each 1990 benchmark country was multiplied by the *value* of real consumption for that country—*not* the PPP.

[22] The mean absolute log-percentage differences are 0.313 and 0.472, respectively.

process.[23] First, each OECD-specific PPP comparing two EU countries is replaced with the corresponding EU-specific PPP. Second, each OECD-specific PPP comparing an EU country and a non-EU country is adjusted to restore transitivity. Thus there are three distinct PPP concepts embedded within the OECD-EKS results.

6 Concluding Remarks

Gordon (1996, p. 292) notes that the use of PPPs from alternative sources can lead to very different assessments of the relative standards of living of countries. In an expression of the widespread confusion that exists among users of PPP data about this seemingly "fragile state of international ... comparisons," he then goes on to ask the obvious question: "[W]hy [do] the sources differ so much?" There are three essential reasons.

First, different sources calculate the same PPPs in the context of different blocs of countries. An example of this was given above when the OECD-calculated EKS PPPs comparing two EU countries were contrasted with the corresponding EKS PPPs calculated by the author. The differences between them are due (in part) to the fact that the EKS index, like all other multilateral indexes, is bloc specific: The comparison of two EU countries in the context of the EU is conceptually different from the comparison of the same two countries in the broader context of the OECD. Similarly, the GK PPPs calculated by the author differ from the corresponding (GK) PPPs calculated for the PWT because the comparison of two

[23] Such a process is necessary since ρ_{EKS}^{ji} is not invariant to even small changes in the size of the bloc (Diewert, 1986, proposition 8).

OECD countries in the context of the OECD is conceptually different from the comparison of the same two countries in the context of the world as a whole.

Second, different sources build the same aggregates from different baskets of goods and services. For example, Final Consumption of Resident Households consists of 159 basic headings under the OECD's classification and 215 under Eurostat's. This fact reveals an additional dimension of conceptual disparity among the OECD-EKS PPPs since those which compare two EU countries were calculated on the basis of the latter classification while all the others were calculated on the basis of the former.

Third, different sources calculate the same PPPs using different methods of aggregation. Using a new type of difference indicator, the preceding section showed that the choice of one method over another can have a substantial impact on the results obtained. Clearly then, further research on the theoretical basis for international comparisons is needed in order to better inform users about which method is "best" in some relevant sense.

[Table 1]

[Table 2]

[Table 3]

[Table 4]

[Table 5]

[Table 6]

[Figure 1]

[Figure 2]

[Figure 3]

[Figure 4]

Appendix

Proof of theorem 1. By T, for any $l \in \mathcal{K}$,

$$\rho^{lj}(P,X,H)\rho^{jj}(P,X,H) = \rho^{lj}(P,X,H) \; .$$

Thus, by P, $\rho^{jj}(P,X,H) = 1$. Now,

$$\begin{aligned} \rho^{ij}(P,X,H) &= \frac{\rho^{ji}(P,X,H)\rho^{ij}(P,X,H)}{\rho^{ji}(P,X,H)} \text{ , by P} \\ &= \frac{\rho^{jj}(P,X,H)}{\rho^{ji}(P,X,H)} \text{ , by T} \\ &= \frac{1}{\rho^{ji}(P,X,H)} \text{ , by WI. } \blacksquare \end{aligned}$$

Proof of theorem 2. First, by 4, there is a real number $\Delta_{A,B} \in \text{range}\,\Delta$ which is associated with any two elements A and B of $\mathcal{P}$. Clearly, $\Delta_{A,B} > 0$ if $B \neq A$. If $B = A$ then

$$\begin{aligned} \Delta_{A,A} &= \frac{2}{K(K-1)} \sum_{j=1}^{K-1} \sum_{i=j+1}^{K} \left| \ln\left(\frac{\rho_A^{li}}{\rho_A^{li}}\right) - \ln\left(\frac{\rho_A^{lj}}{\rho_A^{lj}}\right) \right| \\ &= 0 \; . \end{aligned}$$

Next,

$$\begin{aligned} \Delta_{A,B} &= \frac{2}{K(K-1)} \sum_{j=1}^{K-1} \sum_{i=j+1}^{K} \left| \ln\left(\frac{\rho_B^{li}}{\rho_A^{li}}\right) - \ln\left(\frac{\rho_B^{lj}}{\rho_A^{lj}}\right) \right| \\ &= \frac{2}{K(K-1)} \sum_{j=1}^{K-1} \sum_{i=j+1}^{K} \left| \ln\left(\frac{\rho_A^{li}}{\rho_B^{li}}\right) - \ln\left(\frac{\rho_A^{lj}}{\rho_B^{lj}}\right) \right| \\ &= \Delta_{B,A} \; . \end{aligned}$$

Finally, for any $C \in \mathcal{P}$,

$$\begin{aligned}
\Delta_{A,B} &= \tfrac{2}{K(K-1)} \sum_{j=1}^{K-1} \sum_{i=j+1}^{K} \left| \ln\left(\tfrac{\rho_B^{li}}{\rho_A^{li}}\right) - \ln\left(\tfrac{\rho_B^{lj}}{\rho_A^{lj}}\right) \right| \\
&= \tfrac{2}{K(K-1)} \sum_{j=1}^{K-1} \sum_{i=j+1}^{K} \left| \ln\left(\tfrac{\rho_C^{li}}{\rho_A^{li}}\right) + \ln\left(\tfrac{\rho_B^{li}}{\rho_C^{li}}\right) - \ln\left(\tfrac{\rho_C^{lj}}{\rho_A^{lj}}\right) - \ln\left(\tfrac{\rho_B^{lj}}{\rho_C^{lj}}\right) \right| \\
&\leq \tfrac{2}{K(K-1)} \sum_{j=1}^{K-1} \sum_{i=j+1}^{K} \left\{ \left| \ln\left(\tfrac{\rho_C^{li}}{\rho_A^{li}}\right) - \ln\left(\tfrac{\rho_C^{lj}}{\rho_A^{lj}}\right) \right| + \left| \ln\left(\tfrac{\rho_B^{li}}{\rho_C^{li}}\right) - \ln\left(\tfrac{\rho_B^{lj}}{\rho_C^{lj}}\right) \right| \right\} \\
&= \Delta_{A,C} + \Delta_{C,B} \ . \blacksquare
\end{aligned}$$

Proof of theorem 3. By S1,

$$\rho^{ji}(P, X, H) := \frac{H_i \, p^{i\prime} x^i}{H_j \, p^{j\prime} x^j} \frac{\sigma^j(P, X, H)}{\sigma^i(P, X, H)} > 0 \ .$$

Next,

$$\begin{aligned}
\rho^{jl}(P, X, H)\rho^{li}(P, X, H) &:= \frac{H_l \, p^{l\prime} x^l}{H_j \, p^{j\prime} x^j} \frac{\sigma^j(P, X, H)}{\sigma^l(P, X, H)} \frac{H_i \, p^{i\prime} x^i}{H_l \, p^{l\prime} x^l} \frac{\sigma^l(P, X, H)}{\sigma^i(P, X, H)} \\
&= \frac{H_i \, p^{i\prime} x^i}{H_j \, p^{j\prime} x^j} \frac{\sigma^j(P, X, H)}{\sigma^i(P, X, H)} \\
&=: \rho^{ji}(P, X, H) \ .
\end{aligned}$$

Finally, from (5),

$$\begin{aligned}
& \frac{H_j \, p^{j\prime} x^j}{H_i \, p^{i\prime} x^i} \rho^{ji}(P, X, H) \sigma^i(P, X, H) = \sigma^j(P, X, H) \\
\Rightarrow & \sum_{j=1}^{K} \frac{H_j \, p^{j\prime} x^j}{H_i \, p^{i\prime} x^i} \rho^{ji}(P, X, H) \sigma^i(P, X, H) = \sum_{j=1}^{K} \sigma^j(P, X, H) \\
\Leftrightarrow & \ \sigma^i(P, X, H) = \left\{ \sum_{j=1}^{K} \frac{H_j \, p^{j\prime} x^j}{H_i \, p^{i\prime} x^i} \rho^{ji}(P, X, H) \right\}^{-1} \text{, by S1.} \ \blacksquare
\end{aligned}$$

Proof of theorem 4. By (22) and (24),

$$\rho^{ji}(\bar{P}, \bar{X}, H) = \rho^{ji}((\hat{p}^{US})^{-1}P, \beta\hat{p}^{US}X, H) \ ,$$

where $\hat{p}^{US}$ is the $N \times N$ diagonal matrix with $\hat{p}^{US}_{nn} = p^{US}_n$ for all $n \in \mathcal{N}$. The required equivalence follows from setting $\hat{\lambda} := (\hat{p}^{US})^{-1}$. ■

Proof of theorem 5. By (22) and (24),

$$\sigma(\bar{P}, \bar{X}, H) = \sigma((\hat{p}^{US})^{-1}P, \beta\hat{p}^{US}X, H) \ .$$

The required equivalence follows from setting $\hat{\lambda} := (\hat{p}^{US})^{-1}$. ■

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