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Productivity and Quality in Swedish Schools

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1 Introduction

The purpose of this paper is to evaluate quality and productivity change in the Swedish primary and secondary school system. Both quality and productivity changes are defined here in terms of distance functions, consistent with the Malmquist productivity index introduced by Caves, Christensen and Diewert (1982). We follow Färe, Grosskopf, Lindgren and Roos (1989, 1995) and compute this index 'directly' by applying linear programming techniques to compute the underlying distance functions. One of the key advantages of computing the Malmquist index as opposed to say the Törnqvist index, is that the former does not require data on prices or shares. This is obviously useful in the public sector education context addressed here, where there are no obvious prices for school outputs.

Earlier work using this data set employed econometric techniques to estimate cost and production functions to determine the effect of quality characteristics on the production and costs of education. Heshmati (1996) uses both a hedonic price and index number approach to including quality characteristics in the estimation of production and cost functions for the data set we use here. Heshmati and Kumbhakar (forthcoming) estimate stochastic frontier production and cost functions to predict efficiency for each municipality, which they find to be on the order of 90% to 92% on average.

Here we take an index number approach rather than an econometric approach. We follow Fixler and Zieschang (1992) and augment the Malmquist index to include quality characteristics, which was also the point of departure in Färe, Grosskopf and Roos (1995). Here we generalize our earlier model to allow for both input and output quality characteristics. We also show what conditions are required to allow separate identification of indexes of these quality characteristics and the remaining input and output quantities (generalized inverse homotheticity). We compute quality augmented productivity for a sample of 286 Swedish municipalities over the 1992-1995 period. We find that quality 'matters'.

2 The Productivity Index

The general form of productivity index employed here is constructed from distance functions. The general idea of using distance functions as the basis for an index number is attributed to Malmquist (1953) who used a ratio of input distance functions to define an input quantity index in the consumer context. Later, Caves, Christensen and Diewert (1982) proposed a 'theoretical' productivity index based on distance functions which they dubbed Malmquist productivity indexes. Färe, Grosskopf, Lindgren and Roos first showed in 1989 (the paper was published in 1994) how to directly operationalize a CCD type index. They computed a geometric mean form of that index, which is the basis of what we do here.

Going back to the basic notion of a distance function, we follow Shephard (1970) and define an output distance function for inputs $x^t = (x_1^t, \ldots, x_N^t) \in \Re_+^N$ and outputs by $y^t = (y_1^t, \ldots, y_M^t) \in \Re_+^M$, relative to the output set $P^t(x^t) = \{y^t : x^t \text{ can produce } y^t)\}$ as

$$D_o^t(x^t, y^t) \; = \; \inf\{\theta : (x^t, y^t/\theta) \in \; P^t(x^t)\}. \tag{1}$$

The original productivity indexes proposed by CCD for the output case are

$$M(t) \; = \; D_o^t(x^{t+1}, y^{t+1})/D_o^t(x^t, y^t), \tag{2}$$

and

$$M(t+1) \; = \; D_o^{t+1}(x^{t+1}, y^{t+1})/D_o^{t+1}(x^t, y^t). \tag{3}$$

The form we use here is the geometric mean of $M(t)$ and $M(t+1)$, i.e.,

$$M(t, t+1) \; = \; \left. \frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)} \frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^t, y^t)} \right|^{1/2}. \tag{4}$$

We include quality characteristics by augmenting the input and output vectors to include them specifically. This is the general approach taken in Fixler and Zieschang (1992).[1] Specifically, we can rewrite our distance function for period t as

$$DQ_o^t(x^t, a^t, b^t, y^t) \; = \; \inf\{\theta : (x^t, a^t, (b^t, y^t)/\theta) \in \; PQ^t(x^t, a^t)\}, \tag{5}$$

where a^t and b^t represent input quality attributes and output quality attributes, respectively.[2]The augmented output set $PQ^t(x^t, a^t)$ is the set of all (y^t, b^t) that are producible by (x^t, a^t). Our quality augmented productivity index becomes

[1]They ultimately compute a Törnqvist type index with quality attributes, rather than a Malmquist index.

[2]Fixler and Zieschang suggest that one may also have process quality attributes as well.

$$MQ(t,t+1) \; = \; \frac{DQ_o^t(x^{t+1},a^{t+1},b^{t+1},y^{t+1})}{DQ_o^t(x^t,a^t,b^t,y^t)}\frac{DQ_o^{t+1}(x^{t+1},a^{t+1},b^{t+1},y^{t+1})}{DQ_o^{t+1}(x^t,a^t,b^t,y^t)}\Bigg|^{1/2}. \tag{6}$$

Using these two indexes we can construct an implicit quality index as

$$Q(t,t+1) \; = \; MQ(t,t+1)/M(t,t+1). \tag{7}$$

Following Färe, Grosskopf and Roos (1994), if we assume that the distance functions are separable between input/output quantities and their attributes, as in

$$DQ_o(x,a,b,y) = A(a,b)D_o(x,y) \tag{8}$$

we find that the quality index in (7) simplifies and is of the form

$$Q(t,t+1) \; = \; \left(\frac{A^t(a^{t+1},b^{t+1})}{A^t(a,^t,b^t)}\frac{A^{t+1}(a^{t+1},b^{t+1})}{A^{t+1}(a,^t,b^t)}\right)^{1/2}. \tag{9}$$

This means that we have

$$MQ(t,t+1) \; = \; Q(t,t+1) * M(t,t+1), \tag{10}$$

i.e., productivity change between t and $t+1$ (including quality attributes) i s equal to a pure quality change component and a productivity change component which does not include quality.[3]

A second decomposition of the quality adjusted productivity index (6) is possible if we appeal to a generalization of Shephard's (1970) notion of inverse homotheticity. In order to do so we need to introduce four 'subvector' distance functions, namely

$$D_x^t(x^t,a^t,b^t,y^t) \; = \; \sup\{\lambda : DQ_o^t(\frac{x^t}{\lambda},a^t,b^t,y^t) \leqq 1\}, \tag{11}$$

$$D_a^t(x^t,a^t,b^t,y^t) \; = \; \sup\{\lambda : DQ_o^t(x^t,\frac{a^t}{\lambda},b^t,y^t) \leqq 1\}, \tag{12}$$

$$D_b^t(x^t,a^t,b^t,y^t) \; = \; \inf\{\theta : DQ_o^t(x^t,a^t,\frac{b^t}{\theta},y^t) \leqq 1\}, \tag{13}$$

$$D_y^t(x^t,a^t,b^t,y^t) \; = \; \inf\{\theta : DQ_o^t(x^t,a^t,b^t,\frac{y^t}{0a}) \leqq 1\}. \tag{14}$$

[3]The decomposition in (10) also holds for the original implicit definition of quality in (7). However, that quality component depends on both quality and quantity of inputs and outputs.

Each of these four distance functions are defined on a subvector of inputs and o utputs, respectively, where we are thinking of inputs as (x^t, a^t) and outputs as (b^t, y^t).

We say that the constant returns to scale technology $PQ^t(x^t, a^t)$ is generalized inversely homothetic if the output distance function in (5) can be written as

$$DQ_o^t(x^t, a^t, b^t, y^t) = \left(\frac{D_b^t(1,1,b^t,1)}{D_x^t(x^t,1,1,1)} \frac{D_y^t(1,1,1,y^t)}{D_a^t(1,a^t,1,1)} \right)^{1/2}. \quad (15)$$

Since each of the subvector distance functions are homogeneous of degree +1 in their subvector, DQ_o^t is homogeneous of degree +1 in (b^t, y^t) as it should be by definition of an output distance function. It is homogeneous of degree −1 in (x^t, a^t), due to constant returns to scale. Under generalized homotheticity, the four subvector distance functions have the forms

$$D_x^t(x^t, a^t, b^t, y^t) = \frac{D_x^t(x^t,1,1,1)}{D_b^t(1,1,b^t,1)} \frac{D_a^t(1,a^t,1,1)}{D_y^t(1,1,1,y^t)} \quad (16)$$

$$D_a^t(x^t, a^t, b^t, y^t) = \frac{D_x^t(x^t,1,1,1)}{D_b^t(1,1,b^t,1)} \frac{D_a^t(1,a^t,1,1)}{D_y^t(1,1,1,y^t)} \quad (17)$$

$$D_b^t(x^t, a^t, b^t, y^t) = \frac{D_b^t(1,1,b^t,1)}{D_x^t(x^t,1,1,1)} \frac{D_y^t(1,1,1,y^t)}{D_a^t(1,a^t,1,1)}. \quad (18)$$

$$D_y^t(x^t, a^t, b^t, y^t) = \frac{D_b^t(1,1,b^t,1)}{D_x^t(x^t,1,1,1)} \frac{D_y^t(1,1,1,y^t)}{D_a^t(1,a^t,1,1)}. \quad (19)$$

Inserting (15) into the quality adjusted productivity index we obtain

$$\begin{aligned} MQ(t,t+1) \quad = \quad & \left(\frac{D_b^t(1,1,b^{t+1},1)}{D_x^t(x^{t+1},1,1,1)} \frac{D_y^t(1,1,1,y^{t+1})}{D_a^t(1,a^{t+1},1,1)} \right)^{1/2} \\ & \cdot \left(\frac{D_b^t(1,1,b^t,1)}{D_x^t(x^t,1,1,1)} \frac{D_y^t(1,1,1,y^t)}{D_a^t(1,a^t,1,1)} \right)^{-1/2} \\ & \cdot \left(\frac{D_b^{t+1}(1,1,b^{t+1},1)}{D_x^{t+1}(x^{t+1},1,1,1)} \frac{D_y^{t+1}(1,1,1,y^{t+1})}{D_a^{t+1}(1,a^{t+1},1,1)} ht)^{1/2} \right. \\ & \cdot \left(\frac{D_b^{t+1}(1,1,b^t,1)}{D_x^{t+1}(x^t,1,1,1)} \frac{D_y^{t+1}(1,1,1,y^t)}{D_a^{t+1}(1,a^t,1,1)} \right)^{-1/2^{1/2}}. \end{aligned} \quad (21)$$

If we gather up distance functions according to the subvector involved we arrive at the equivalent expression

$$MQ(t,t{+}1) = \frac{\left(\frac{D_b^t(1,1,b^{t+1},1)}{D_b^t(1,1,b^t,1)}\frac{D_b^{t+1}(1,1,b^{t+1},1)}{D_b^{t+1}(1,1,b^t,1)}\right)^{1/2}\left(\frac{D_y^t(1,1,1,y^{t+1})}{D_y^t(1,1,1,y^t)}\frac{D_y^{t+1}(1,1,1,yt+1}{D_y^{t+1}(1,1,1,y^t)}\right)^{1/2}}{\left(\frac{D_x^t(x^{t+1},1,1,1}{D_x^t(x^t,1,1,1)}\frac{D_x^{t+1}(x^{t+1},1,1,1}{D_x^{t+1}(x^t,1,1,1)}\right)^{1/2}\left(\frac{D_a^t(1,a^{t+1},1,1)}{D_a^t(1,a^t,1,1)}\frac{D_a^{t+1}(1,a^{t+1},1,1)}{D_a^{t+1}(1,a^t,1,1)}\right)^{1/2}}\Bigg|^{1/2} \tag{22}$$

Thus the quality adjusted productivity index under generalized inverse homotheticity consits of four quantity indexes: one for each argument in the quality adjusted distance function. This formulation allows us to identify changes in overall quality adjusted productivity due to four components: change in quantity of x, y, b and a. We note that these component quantity indexes together form a productivity index which is of the form Diewert dubs Hicks-Moorsteen; it is constructed as ratios of output quantity indexes to input quantity indexes. As such this index and its decomposition is the Malmquist analog of the Törnqvist decomposition in Fixler and Zieschang (1992).

3 Computation

We follow Färe, Grosskopf, Lindgren and Roos (1994) and compute the component distance functions using activity analysis. This is the same technique widely used in data envelopment analysis to compute technical efficiency.

An example of how we compute one of the quality augmented distance functions is

$$\begin{aligned}
(DQ_o^t(x^{k',t+1}, a^{k',t+1}, b^{k',t+1}, y^{k',t+1}))^{-1} &= \max_{z,\theta} \theta && (23)\\
s.t. & \\
\sum_{k=1}^{K} z_k y_{km}^t &\geqq \theta y_{k'm}^{t+1}, m = 1, \ldots, M, \\
mber \sum_{k=1}^{K} z_k b_{ki}^t &\geqq \theta b_{k'i}^{t+1}, i = 1, \ldots, B, && (24)\\
mber \sum_{k=1}^{K} z_k x_{kn}^t &\leqq x_{k'n}^{t+1}, n = 1, \ldots, N, && (25)\\
\sum_{k=1}^{K} z_k a_{kj}^t &\leqq a_{k'j}^{t+1}, j = 1, \ldots, A, \\
z_k &\geqq 0, k = 1, \ldots, K.
\end{aligned}$$

Observations are indexed by $k = 1, \ldots, K$, and the linear programming problem above simultaneously constructs a reference technology as convex cone combinations of observed data (the intensity variables, z_k, serve this purpose) and computes the value of the distance function for observation k'.[4] In this particular case, the reference technology is constructed from all the observations in the sample from period t. Each observation ($k' = 1, \ldots, K$) is evaluated relative to this frontier; in this case these observations are from period $t+1$. This is sometimes referred to as a mixed period problem. One can compute $DQ_o^t(x^t, a^t, b^t, y^t)$, a single period problem, by substituting values from period t on the right-hand side of the inequalities.

The distance functions for the $M(t, t+1)$ index are computed similarly, except that the constraints with a and b are not included.

The subvector distance functions used to compute the quality adjusted Malmquist productivity index under gneralized inverse homotheticity are also computed as above. As an example, we take $D_b^t(1, 1, b^{t+1}, 1)$ which is computed for each observation ($k' = 1, \ldots, K$) as

$$
\begin{aligned}
(D_b^t(1,1,b^{k',t+1},1))^{-1} &= \max_{z,\theta} \theta && (26)\\
s.t. & \\
\sum_{k=1}^{K} z_k y_{km}^t / \hat{y}_m^t &\geqq 1, m = 1, \ldots, M, \\
\sum_{k=1}^{K} z_k b_{ki}^t &\geqq \theta b_{k'i}^{t+1}, i = 1, \ldots, B, \\
mber \sum_{k=1}^{K} z_k x_{kn}^t / \hat{x}_n^t &\leqq 1, n = 1, \ldots, N, && (27)\\
\sum_{k=1}^{K} z_k a_{kj}^t / \hat{a}_j^t &\leqq 1, j = 1, \ldots, A, \\
z_k &\geqq 0, k = 1, \ldots, K.
\end{aligned}
$$

Note that the left-hand side of the constraints in this problem are slightly different than those in (23): the data are normalized by a given value of the relevant variable, which is denoted with a 'hat', for all of the constraints *except* the subvector being scaled. The right-hand sides, look quite different; all of the constants on the right-hand side are equal to one except for the constraint associated with the subvector being scaled–in this case b. The values for the normalization are chosen by finding data that is 'efficient' in a nonnormalized version of (24).

[4]For more details on the properties which the technology satisfies, see Färe, Grosskopf and Lovell (1994).

4 The Data

The data consists of the primary and secondary school services provided by th e entire population of 286 Swedish municipalities during the 1992/3-1994/5 school years. On behalf of the National Board of School Education, the data has been collected and proc essed by Statistics Sweden (SCB) to provide municipalities with various measures of perf ormance. These include the issues of organization, teaching, resource-use, and performan ce for different schools and types of education. The intensive controls performed foll owing the data collection reduced possible measurement errors. The data are thus consider ed by the SCB to be of relatively high quality. The variables used in this study consist of input and output data as well as a number of variables representing the quality of input and output and municipality characteristics variables.

Output is defined as the number of students and as the product of the number of students and the average teaching hours per student at the primary and seconda ry schools. The number of teaching hours differ among municipalities due to the difference in the composition and teacher densities of different stages of the primary and second ary schools. The number of students is an aggregate number of both resident and non-resident students in the respective municipality. The inputs include both input quantities: namel y number of teachers and space[5], as well as expenses related to teaching, materials, library, school meals, health care, counseling, administration, complementary education, rent and busing.[6] The available output quality indicators include the number of students with complete grades, average grades and transition to high school. The input quality indicators include space per student, trained teachers, the numbe r of hours teachers are in attendance and teacher density. Variables that were ultimately included in our model specification (discussed below) are indicated with asterisks.

The other municipality characteristics include the student density in schools and class rooms, non-Swedish citizens, Swedish as second language, native language, level s of education and income of the families, young and total population, per capita ta x, general state support, distance measure, and private schools. A number of qualitative v ariables related to the types of policy, renting practices and organizations are introdu ced. The policy variables indicate whether the municipality is governed by a socialistic majori ty, a conservative majority or their coalition. The renting variable indicates whethe r the contracts were based on average cost price, market clearing price, or other types of pric ing practices. The organization variables indicates whether the

[5]The table includes space in square meters per student, our specification uses unadjusted square meters as an input.

[6]We include expenditures on li braries, counseling and complementary education as input quality variables in our model specification. We use the number of teachers and square meters of space rather than expenditures on teachers and rent in our specification.

organization of the municipali ty has changed very much, moderately, or not at all. Change is defined in terms of the existence of competitive environment considering contracting out activities. Summary stat istics of the data are given in Table 1.

The teaching cost variable refers to costs of teachers' salary including payr oll taxes. Material cost is defined as the sum of expenses related to the purchase of book s, newspapers, printing, computers facilities, equipment, wages for technicians, a nd study related travels. The library variable represents costs to the municipalities as sociated with operating schools libraries including purchases of books and salaries paid to l ibrarians. The school meal measures costs of providing the student with a school meal. It cove rs wages, food, administration and transportation costs. It does not include the rent of spaces and fees paid by students. However, it has been adjusted for certain revenues. The healt h care variable is an aggregate of all expenses in providing school health care servic es including salaries to doctors, nurses, psychologists, advisors, support personnel, as wel l as medicine and insurances. The counseling variable includes all costs associated with diff erent programs informing the students about the issues related to the job market. The administ ration variable is defined as the primary and secondary schools share of the total costs to the municipality associated with the administration of the schools. It includes both local and c entral administration. Different weights have been used in the distribution of the tot al cost among various types of schools within a municipality. The complementary education var iable includes all expenses related to complementary education of both administrative and teaching staff. It also includes costs associated with hiring substitute teachers, trave ling and organizing seminars. The rent variable includes the cost of renting, cleaning, maintenance and the other costs related to the school buildings. Busing (taxi) is defined a s cost of bus (taxi) transportation to the municipality per student.

Student density per classroom (per school) is obtained by dividing the number of students by the number of classrooms (the number of schools) in the municipalit y, respectively. Complete grades are defined as the percent of students who receiv ed incomplete grades (during the spring semester) only in at most two subjects. Av erage grade is defined as the average grade at the end of the last (ninth) year of the seco ndary school at the municipality level. The space variable is measured as the average square me ter space per student. Teacher is defined as the number of full-time equivalent teachers. Tra ined teacher is defined as the number of certificated teachers. Teachers' attendance is meas ured as the percentage of nonabsent hours due to sickness, temporary lay-off or care of chi ld. Promotion is defined as the number students at the ninth grade in the spring se mester, who started the high school program in the subsequent semester.

The non-Swedish citizen variable is the share of population between the ages of 20-64, not holding a Nordic citizenship and who did immigrate to Sweden sometime durin g 1984-1994. The Swedish language variable is defined as the share of students w ith Swedish as their second language. Native language is defined as the share of students atte nding courses in their native language. The low education variable is defined as the percent of population between the ages of 20-64, with at most 9 years of education. Low income is def ined as the percent of municipality population between the ages of 0-17, who are members of households receiving social security benefits. The per capita tax variable is obtained by dividing the total tax revenue by the entire municipality population. The variable POP715, i s defined as the percent of population at the school age of 7-15. Population is the total population residing in the municipality. State support is the sum of general support provided by the state to promote the municipalities production of services. Distance is a measure of housing (population) density within a municipality measured as the average equi-distanc e between two neighboring houses.

The municipalities are divided into nine different groups depending on the si ze of population, the geographic location and the degree of industrialization. This c lassification is used to capture possible municipality heterogeneity. The municipality types include: three largest cities, suburban, large, medium, industrial, rural, thinly populated, o ther large and other minor. All variables expressed in SEK are transformed to fixed 1994 price s using the net municipality cost price index.

5 Model Specification

The model specification was determined to a large extent by the data available, which were discussed above. The unit of observation is the municipality level. This implies that there may be a large number of schools included in one observation.

The available measures of output quantity are fairly crude: the number of students and the total number of hours of instruction. As output quality attributes we include number of students promoted to the next level of education and the average grade on the exam taken in the 9th grade to determine whether the student will be promoted.

On the input side, the quantity of inputs include the number of teachers and total square meters of space. Also included as input ‘quantity’ measures were a number of cost categories. Although we did not aggregate them, we conceptually grouped them as indicated below. Input quality measures included expenditures on several categories (libraries, counseling and complementary education) as well as teacher training. To summarize, we have the following specification

- Output Quantities

  students, total hours of instruction

- Output Qualities

  students promoted, average grade in 9th grade

- Input Quantities

  teachers, square meters of space

  social services: expenditures on meals, health, and busing

  other input 'quantities': expenditures on administration and materials

- Input Qualities

  input quality attribute: number of teachers with extra training.

  other quality-related inputs: expenditures on libraries, counseling and complem entary education for staff

Variables with asterisks in Table 1 were included in the specification.

Clearly there are alternative ways to model schooling in Sweden, even using just these variables. For example, one could argue that the items included in social services could very well be included as services (outputs) provided by schools.

Table 1. Summary statistics of the Swedish primary and secondary schools, 1992/ 3-1994/5

Variable	Mean	St.Dev.	Minimum	Maximum
OUTPUTS:				
Number of students (STUDEN)*	3080.21	4315.62	284.0 0	51200.00
Teaching hours (HOURS)*	3485424.89	4957880.69	325874.26	56667961.17
INPUTS:				
Space per student (SPACE)*	16.48	3.94	6.00	42.73
No of teachers (TEACHE)*	255.59	353.64	29.65	4181.02
Teaching (TEACHI)	25161.27	2972.47	17900.00	431 28.96
Material (MATERI)*	1903.11	495.98	800.00	4 154.00
School meal (MEAL)*	3106.02	699.78	1268.50	66 00.00
School health care (HEALTH)*	1506.48	516.29	405. 27	4863.22
Administration (ADMINI)*	3634.95	905.76	1500.00	9017.22
Rent (RENT)	11935.46	3008.79	4700.00	2462 0.06
Busing (BUSING)*	2083.34	1081.47	40.00	56 30.00
Total cost/student (TOTCOS)	50336.54	6298.04	35900.0 0	87933.13
OUTPUT QUALITY VARIABLES:				
Average grade (GRADE)*	3.19	0.09	2.91	3.61
Passed (PASS)	2996.94	4094.55	274.54	4768 7.68
Promoted (PROMOT)*	2992.51	4128.89	271.95	49 331.20
INPUT QUALITY VARIABLES:				
No of trained teachers (TRAINE)*	240.47	331.57	24. 53	3767.10
100.00				
Library (LIBRAR)*	196.40	120.46	0.00	1035.9 4
Counseling (COUNSE)*	280.24	105.06	81.05	80 0.00
Complementary edu. (COMPLE)*	529.29	210.83	74.00	1960.00
No of class rooms (ROOMS)	139.82	186.57	18.39	2165.82
No of schools (SCHOOL)	16.30	17.44	3.00	169. 00
MUNICIPALITY CHAR. VARIABLES:				
% independent schools (PRIVAT)	0.65	1.28	0.00	7.80
Population 7-16 years old (POP715)	10.96	1.07	6.58	15.00
Municipality popul. (POPULA)	29798.01	50485.12	2840.00	643216.08
% non-Swedish sitizen (NONSWE)	2.35	1.42	0.00	10.00
% attending Sw2 classes (SWE2ND)	4.29	4.71	0.00	38.42
% attending native lang. (NATIVE)	3.73	5.19	0.00	38.70
% low education (LOWEDU)	31.99	6.57	8.13	48.00
% low income (LOWINC)	10.82	3.73	0.00	29.54
Tax per capita (TAXCAP)	891.19	133.44	638.30	1617.34
Total state support (STATE)	242054.83	405584.80	18675. 79	6498985.20
Distance (DISTAN)	273.02	261.19	17.67	2003.00

Table 1. (cont.) Summary statistics, 1992/3-1994/5

Variable	Mean	St.Dev.	Minimum	Maximum
RENTING PRACTICES:				
Average cost pricing (COSTP)	0.56	0.50	0.00	1.00
Market pricing (MARKEP)	0.09	0.29	0.00	1.00
Other pricing (OTHERP)	0.34	0.48	0.00	1.00
POLICY VARIABLES:				
Conservative majority (CONSER)	0.60	0.49	0.00	1.00
Social democratic majority (SOCIAL)	0.25	0.44	0.00	1.00
Unclear majority (UNCLEA)	0.15	0.35	0.00	1.00
ORGANIZATIONAL VARIABLES:				
Very much changed (HOT)	0.13	0.34	0.00	1.00
Minor changed (SEMI)	0.38	0.49	0.00	1.00
No changes (COLD)	0.36	0.48	0.00	1.00

No of observation is: 3 x 286 = 858.

6 Results

We first compute the simple Malmquist productivity indexes with and without quality characteristics as in (4) and (6) and use these to construct the quality index as the ratio of these two as in (7). These are computed for each municipality in our sample for the period 1992-93 and 1993-94, the years for which we have data.

We begin with summary statistics for these results which are included in Table 2. These averages suggest that there have been small improvements in performance on average over the 1992-94 time period. The first time period included a measured improvement in quality on average, whereas the second time period (1993-94) suggested no real change in quality on average. We note that by construction, $MQ(t, t+1) = M(t, t+1) * Q(t, t+1)$, thus one can identify sources of overall productivity change (including quality) : change in productivity which excludes quality factors and (residual) changes which are due to changes in quality. In this formulation we have not imposed separability or inverse homotheticity, thus the quality change component is computed indirectly as $QM(t, t+1)/M(t, t+1)$.

Table 2. Summary statistics of Results, 1992/3-1994/5

1992-93					
Variable	Mean	Median	Min	Max	St.Dev.
QM(t,t+1)	1.0461	1.0282	0.7195	2.3836	0.1390
M(t,t+1)	1.0189	1.0206	0.7764	1.2743	0.0676
Q(t,t+1)	1.0274	1.0069	0.6933	2.3202	0.1209
No of observations: 286.					

1993-94					
Variable	Mean	Median	Min	Max	St.Dev.
QM(t,t+1)	1.0236	1.0188	0.6445	1.6367	0.0927
M(t,t+1)	1.0264	1.0275	0.7850	1.9261	0.0930
Q(t,t+1)	0.9991	1.0001	0.6290	1.5142	0.0745
No. of observations: 286.					

Of course, averages may mask more complex underlying patterns. In Table 3 we include averages and frequencies broken out by whether the municipality realized progress (values of the indexes greater than one) or regress (values less than one). Here we see that the frequency of progress versus regress declined slightly over the 1992-94 time period, although progress in all three indexes was more frequent than regress.

We also performed a battery of nonparametric tests to determine whether this average progress is significant. The null hypothesis is that the index in the given period is equal to one. We include the results for three of these: ANOVA, median test and Kolmogorov-Smirnov test. The first two tell us about location or moments, whereas the last is a test based on the empirical distribution function. The results are summarized in Table 4. All except one of these statistics (ANOVA for Q(t,t+1) for 1993-94) suggest that the null hypothesis of no change is rejected in favor of progress.

Table 3. Average Progress versus Regress, 1992/3-1993/4

Variable	1992-93 Progress (count > 1)	1992-93 Regress (count < 1)	1993-94 Progress (count >1)	1993-94 Regress (count < 1)
QM(t,t+1)	1.094	0.953	1.069	0.946
	(189)	(97)	(181)	(105)
M(t,t+1)	1.055	0.951	1.063	0.961
	(187)	(99)	(183)	(103)
Q(t,t+1)	1.067	0.968	1.038	0.959
	(171)	(115)	(144)	(142)

Table 4. Nonparametric Tests: $H_0 : Index = 1$

INDEX	ANOVA (F)	MEDIAN (z)	KOL.-SMIRNOV (KSA)
QM(t,t+1)			
1992-93	31.44	16.76	7.88
1993-94	16.69	-16.24	7.56
M(t,t+1)			
1992-93	22.26	16.63	7.80
1993-94	22.99	-16.37	7.65
Q(t,t+1)			
1992-93	14.69	15.64	7.17
1993-94	0.05	-13.84	6.02

We also correlated the indexes by year with some municipal characteristics, including the teacher-pupil ratio, total expenditures per student, the percent of the municipal population that is not Swedish, the percent of students who are taught in a language other than Swedish (% native), the percent of the adult population classified as having low educational attainment, the percent of the population classified as low income, the tax rate, grants, and a measure of distance. These are displayed Table 5. For the 1992-93 period, there are no significant correlations between the productivity measures and the characteristics included in the table, with the exception of the percent non-Swedish and percent taught in a language other than Swedish: these are positively correlated with the unadjusted productivity index. We find more evidence of correlation with the results from 1993-94. Interestingly, the quality index is positively correlated with the teacher pupil ratio and the percent classified as low educational attainment. Higher taxes are negatively correlated with quality, however there is a weak negative correlation between taxes and per pupil expenditures as well.

Table 5. Correlation Coefficients, by year

Variable	QM(92,93)	M(92,93)	Q(92,93)	QM(93,94)	M(93,94)	Q(93,94)
Teach/Pup	0.081	0.095	0.042	0.094	0.019	0.105
	(.172)	(.109)	(.483)	(.114)	(.751)	(.078)
exp/pup	0.062	0.078	0.029	0.061	0.047	0.022
	(.295)	(.189)	(.624)	(.311)	(.424)	(.707)
% nonSwede	0.009	0.111	-0.042	-0.021	0.051	-0.084
	(.882)	(.060)	(.374)	(728)	(389)	(.157)
% native lang	0.054	0.101	0.0001	-0.052	0.015	-0.085
	(.367)	(.088)	(.998)	(.386)	(.807)	(.151)
% low ed	0.039	-0.019	0.057	0.078	0.009	0.100
	(.510)	(.749)	(.334)	(.187)	(.874)	(.092)
% low inc	0.018	0.051	-0.006	0.034	0.011	0.043
	((.767)	(.391)	(.918)	(.571)	(.856)	(.470)
tax	-0.004	-0.035	0.016	-0.047	0.061	-0.141
	(.946)	(.552)	(.793)	(.432)	(.300)	(.017)
aid	-0.019	0.017	-0.031	-0.047	-0.054	-.002
	(.750)	(.779)	(.600)	(.433)	(.362)	(.973)
dist	0.005	-0.061	0.041	-0.003	-0.063	0.077
	(.935)	(.306)	(.488)	(.965)	(.287)	(.194)

7 Summary

The purpose of this paper was to model and compute productivity, including a measure of quality, for Swedish primary and secondary schools. At the modelling level, we show how to include quality characteristics in the Malmquist productivity index in a way that is analogous to the approach taken by Fixler and Zieschang for the Törnqvist index, i.e., we develop the form of the Malmquist index required for a complete decomposition of quality into separate components. This requires inverse homotheticity of the underlying technology, which is quite restrictive. In our application in this paper we compute productivity without this restriction, yielding measures of quality augmented productivity and an aggregate quality index.

Our computational approach exploits the relationship between distance functions (which are the building blocks of the Malmquist productivity index) and Farrell measures of technical efficiency, which may be computed using simple linear programming techniques. By directly estimating distance functions, which do not require information on prices or shares, we can directly compute productivity for a public service where prices are typically unavailable. This approach also accommodates the specification of multiple outputs, and as we show, quality characteristcs.

We apply our approach to data on Swedish primary and secondary schools operating in 1992-94. The data available implied that our specification is restricted to what might be called an intermediate production model, i.e., we have data on various input, quality, and output categories for this time period. Our outputs are restricted to data on hours taught, promotion and standardized exam scores. Thus we do not have panel data which would allow us to look at post school performance as part of the outputs of the schooling process. Nor do we have cohort data required to compute value-added type outputs (improvement in standardized scores controlling for previous performance and fixed student and parent characteristics) favored for example by Hanushek (1986). Nevertheless, we would argue that our application does provide valuable information on performance for the schools (municipalities) in our sample.

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