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Issues in Measuring Price Changes of Rent of Shelter

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1. Introduction

Shelter is the largest single component of consumer spending, comprising 27 percent of the weight of the U.S. Consumer Price Index (CPI). Similarly, shelter represents about 15 percent of Personal Consumption Expenditures and about 10 percent of Gross Domestic Product. Rental housing is included in the aggregate U.S. productivity statistics, and rented structures are included as an input in the multifactor productivity measures. Because of its large weight, even small biases in measuring price change for shelter potentially have large effects on measurement of inflation, output, or productivity.

Shelter has a number of distinguishing features: extremely long life (the median age of housing units in the United States in 1993 was 28 years), heterogeneity, spatial fixity, and special tax treatment. (A survey of research on housing markets is Smith, Rosen, and Fallis, 1988.) The U.S. Bureau of Labor Statistics (BLS) has made a number of changes to address biases in the shelter indexes: adoption of rental equivalence in 1983, introduction of depreciation adjustment in 1988, and changes in the imputation of owners' equivalent rent (OER) in 1995 and 1996.

The recent release of the final report of the Advisory Commission to Study the Consumer Price Index (U.S. Senate, 1996) has focused new attention on adjustment for quality change in the shelter components, as well as other components of the CPI. In particular, the Commission estimated that failure to fully account for quality improvements results in an upward bias of 0.25 percent per year for the rent and OER indexes.

Section 2 of this paper will discuss the adjustments made by the BLS for quality and depreciation, including a critique of the shelter section of the Advisory Commission's final report. Section 3 discusses the imputation of OER and presents research that was conducted prior to changes made by the BLS to its OER index in 1995 and 1996.

2. Adjustment for Quality and Depreciation

2.1 BLS Methods for Quality Adjustment

There are a number of issues involved in adjusting the rent and OER indexes for quality change that are described in greater detail in U.S. Department of Labor (1992). In this section I will provide a short description of the most important issues. I look specifically at (a) improvements in the stock of housing occurring through the entry of newly constructed units, (b) accounting for physical depreciation as a housing unit ages, (c) accounting for renovations and other improvements to a housing unit, and (d) accounting for changes in terms of the rental contract, especially changes in the allocation of utilities between tenant and landlord.

Perhaps the most important method of adjusting for quality change of the housing stock occurs through the addition of new construction to the sample. Each year a sample of newly constructed units is added to the CPI sample. The sample augmentation method used to bring in new construction is analogous to the “overlap” method (see Moulton and Smedley, 1997) in that it permits the average rent level of the sample to rise without being reflected as a price increase. In other words, the difference between the rent of the new unit and the average rent of the existing units is taken as a measure of the difference in quality between the new unit and the average of the existing units. This difference can be significant—according to the American Housing Survey, in 1993 the median monthly housing cost of rental units constructed within the previous 4 years was $623, compared to $487 for all units. In the case of housing, the facts that new construction is typically a very small percentage (1 to 3 percent) of the housing stock and that there is significant competition between landlords in rental markets suggest that the sample augmentation method will produce reasonably accurate adjustments for the quality of new units.

It should not be surprising that landlords do not maintain housing units perfectly, so that the level of service provided by the unit tends to decline as the unit ages. Until 1988, the CPI missed this depreciation, which imparted a downward bias to the rent index (Ozanne, 1981). Beginning in 1988, the BLS began adjusting rent changes for physical depreciation based upon hedonic estimates that account for age and vintage effects (Randolph, 1988b). The estimates of downward bias were 0.36

percent per year for the rent index and 0.41 percent per year for the OER index. I will discuss Randolph's method and other issues raised by the CPI Advisory Commission related to depreciation in section 2.3.

Since 1989 the BLS has used hedonic methods to account for major renovations that result in changes to the structure, such as addition of central air conditioning or a bathroom. These coefficients are by-products of the hedonic analysis conducted to estimate the depreciation factor. Minor, recurring improvements (painting, replacement of carpet or appliances, etc.) are usually treated by the BLS as part of maintenance and do not result in quality adjustment. In a few cases, a major quality change will be adjusted by the link method if an appropriate hedonic estimate is not available.

Finally, direct quality adjustments are made for addition of new appliances or shifts in contractual arrangements for payments of utilities. These adjustments are made using estimates of the value of the change in the unit or the shift in payment. See U.S. Department of Labor (1992) for details.

2.2 The CPI Advisory Commission's Estimate of Bias

The Advisory Commission's report begins its discussion of quality in housing by pointing to an interesting discrepancy by the time trend of two measures of price. From 1976 to 1993 the median monthly cost of rental housing increased from $167 to $487, an increase of 192 percent, while over the same period the CPI rent index increased 146 percent. The annualized difference between the growth rates of these two series is 1.0 percent per year. The Commission's report then attempts to account for the divergence of these two series by changes in the size and characteristics of housing. After reporting a number of statistics on various attributes of quality of housing, they present their calculation of bias:

> "To conclude that the CPI is unbiased, we would have to determine that the quality of the average rental unit increased by 1.0 percent per year over that period, or 18 percent over the entire period. From the evidence we have examined, we believe that 20 percent is a low-end estimate of the increase in the average size of apartments, which would support the conclusion that the average rent per square foot has increased no faster than the CPI. But also, we find convincing evidence that the

> average quality of apartments per square foot has increased as well. The transition to central air conditioning proceeded at a rapid rate during the past two decades. Other amenities were added which increased the average quality of apartments, particularly swimming pools, health clubs, on-site free parking, and climate (since the mix of apartments shifted toward southern climates which reduced the impact of winter weather on tenants, particularly older tenants).
>
> "For the period since 1970 we find it plausible that the CPI accurately measures rent per square foot of apartment space, but its measure of shelter rent is upward biased by neglecting the increase in the quality of apartments per square foot...A conservative estimate is that the total increase in apartment quality per square foot, including the rental value of all appliances, central air conditioning, and improved bathroom plumbing, and other amenities, amounts to 10 percent over the past 40 years, or 0.25 percent a year. Accordingly Table 2 records an upward bias in the CPI of 0.25 percent per year for the shelter component, and this may well be an understatement." (U.S. Senate, 1996, pp. 30-31)

The Advisory Commission apparently begins with the assumption that any difference between the median monthly cost of housing (from the American Housing Survey) and the CPI rent index is wholly attributable to quality changes. Yet there are a number of other important factors that affected trends of these two indexes which need to be taken into account.

(a) The methods used to construct the CPI rent index have changed over time. Beginning in 1988 hedonic-based methods were introduced to adjust for physical depreciation of rental units (Randolph 1988b), which increased the growth rate of the index by about 0.36 percent per year. In 1995 the estimator was changed to avoid a bias caused by under-reporting by survey respondents to a recall question about rent changes during the previous month. The effect of this change was to increase the growth rate of the index by about 0.22 percent per year (Armknecht, Moulton, and Stewart, 1995). Whether or not the Commission's agrees with these changes, if their intention is to measure bias relative to the procedures that are currently used by the BLS, then they should adjust for the effects of these changes on the historical data.

(b) The CPI rent index holds the geographical distribution of the population fixed by combining the 44 area indexes into an all-areas index using a fixed weight formula. The median housing cost is influenced by migration and changes in the distribution of housing between renters and owners within region, factors which do not affect the CPI rent index.

(c) The CPI measures the change of contract rents holding utilities constant. Thus, as described earlier, increases in cost of utilities affect the rent index in proportion to the weight of the units with utilities included in their rent. (For the units where the utilities are not included in the rent, the weight in the CPI is carried directly by the utility indexes.) On the other hand, the median monthly housing cost from the American Housing Survey (AHS) includes "contract rent plus the estimated average monthly cost of utilities (electricity, gas, water, and sewage disposal), and fuels (oil, coal, kerosene, wood, etc.); property insurance, mobile home land rent, and garbage and trash collection…" (U.S. Department of Commerce, 1995, p. A-19). So the median monthly housing cost includes major utilities regardless of the allocation of payment between landlord and tenant.

Column (1) of Table 1 presents the CPI for rent for selected years during 1976-93. In column (2) I adjust the CPI to make it more comparable to the AHS median housing cost by (a) adjusting the pre-1995 index for the recall bias, (b) adjusting the pre-1988 data for Randolph's estimate of depreciation bias, and (c) combining the adjusted rent data with price indexes for fuel oil, electricity, and piped gas, with weights drawn from the expenditures of renters reported in the 1972-73 Consumer Expenditure Survey. Adjustments (a) and (b) increased the growth rate of the adjusted index relative to the unadjusted published index, whereas adjustment (c) had little effect because cumulative utility inflation was similar to cumulative rent inflation over this period (electricity and fuel oil had smaller total price change than adjusted rent over this period, while piped gas had larger total price change).

Column (3) of Table 1 presents the AHS median monthly housing cost and column (4) adjusts for migration by taking a weighted average of changes in median monthly housing costs in each of the four major regions, holding the regional distribution of housing units fixed at its 1976 levels. This adjustment had little effect on the overall percentage change.

Comparing columns (2) and (4), we see that the difference in the annualized growth over the 17-year period, 1976-93, is now reduced to 0.69 percent per year. This is the part of the difference that ought to be explained by changes in quality.

How much of that can be accounted for by changes in unit size? Several observers have pointed out that rents generally increase less than proportionally to apartment size, which implies that the Advisory Commission's proportional adjustment for apartment size would overstate the value of the increase.

A more fundamental problem with the Advisory Commission's calculation, however, is that their factual premise---the assertion that average apartment size has increased 20 percent from 1976 to 1993---is clearly wrong. Unfortunately, data are not available that give an exact measure of the growth in size of rental units since 1976. The following points summarize this evidence.

(a) The American Housing Survey has published median size of single detached and mobile home rental units since 1985. As shown in column (5), from 1985 to 1993 the median unit size increased 2.2 percent, an average of less than 0.3 percent per year.[1]

(b) The Energy Department's Residential Energy Consumption Survey collected data on the average square feet of housing units over the somewhat earlier period from 1980 to 1993. Although these data are less reliable than the American Housing Survey, the trend is the same. The Energy Department data show the increase in average size of all rental units from 1980 to 1993 as 3.5 percent, i.e., also about 0.3 percent per year.[2]

(c) In order for the average size of rental units to increase as rapidly as the Advisory Commission claimed, the average size of newly constructed apartments would need to be much larger (roughly 30-40 percent larger) than the average size of existing apartments. This difference in size would be required because new construction in most years represents between 1 to 3 percent of the existing housing stock. Data comparisons indicate that newly constructed apartments, in fact, were not much bigger than existing apartments over this period. As shown in column (14) of Table 1, in 1993 the mean space of a newly constructed multifamily units was 1,065 square feet, whereas

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[1] Source: U. S. Department of Commerce and U.S. Department of Housing and Urban Development, *American Housing Survey for the United States in 1993* (and same publication in 1985), Current Housing Reports H150/93, Table 2-3.

[2] The average floorspace of rental units in 1980 was 1,090 square feet (U.S. Department of Energy, Energy Information Administration, *Housing Characteristics, 1980,* Table 9), and in 1993 was 1,128 square feet (*Housing Characteristics, 1993,* Table 3.4).

according to the Energy Department survey, the mean space of existing of multifamily dwellings was 970 square feet, a difference of about 10 percent.[3]

The median number of rooms increased from 4.0 to 4.2 over the 17-year interval, an increase of 5 percent. As seen in columns (6) to (10), the distribution of structure types shifted toward more single attached, multifamily in buildings with 5 or more units, and mobile homes, and away from single detached and multifamily units in buildings with 2-4 units. These changes shift the distribution of rental units across structure types with differing average sizes, but net effect of these shifts on average floorspace is not obvious—the biggest growth is in multifamily units in buildings with 5 or more units, which are the smallest units, and in single family attached, which are larger on average than other structure types except single family detached. In view of the evidence presented above I conclude that the average floorspace probably increased about 5 to 6 percent over the 17-year period 1976-93, or about 0.3 percent per year, rather than the 20 percent increase claimed by the Advisory Commission. I have not yet located a calculation of the elasticity of floorspace on price that is fully appropriate for the purpose of quality adjustment, but since the elasticity is surely less than 1, I conclude that the increase in average floorspace accounts for between 0.2 to 0.25 percent per year of the differential between the rent index and median housing costs.

The Commission highlighted other improvements as well. As shown in column (11), the percentage of apartments with central air conditioning grew substantially, from 16 percent in 1976 to 35 percent in 1993. The market valuation of this shift can be measured, at least approximately, from hedonic regression coefficients. For example, Kokoski and Moulton (1996) report a coefficient of 12.4 percent for central air conditioning in a semilogarithmic regression estimated using 1989-90 data. Randolph (1988b) reports 10.2 percent for 1983. In another study, Moulton (1995), I found that the coefficient varied across areas and tended to be larger in the South, where central air conditioning is more extensively used. If we raise the coefficient to 16 percent to account for the possibility that renters, especially in the South, may “value these amenities at more than their extra cost” (U.S.

[3] U.S. Department of Energy, Energy Information Administration, *Housing Characteristics, 1993.*

Senate, 1996, p. 29), and then multiply the adjusted coefficient by the 19 percentage point increase in rental units with air conditioning, the increased availability of central air conditioning accounts for about 3 percent or 0.17 percent per year.

Other quality improvements, such as more units with second bathrooms (see column (13)), are expected to have significant, albeit less effect. I will adjust the 9 percentage point increase in rental units with second bathrooms (from 7 percent to 16 percent) by hedonic coefficients of between 8 percent (Randolph, 1988b) to 13 percent (Kokoski and Moulton, 1996), adding a net change of 0.04 percent to 0.07 percent per year. Other quality improvements, such as improved wiring, plumbing, windows, improved quality of appliances, and other amenities, would not be fully captured by regression methods. It seems reasonable, though, to expect that the cumulative effects of these changes is smaller than the quantifiable changes in central air conditioning and second bathrooms, so that they may account for at most another 0.05 percent per year. Reviewing all of these non-floorspace sources of quality improvement, it appears to me that the Advisory Commission's estimate of 0.25 percent per year may have been conservative, and a range of 0.25 to 0.30 percent per year appears to me be a reasonable estimate of quality change.

Adding together the effects of floorspace and non-floorspace these two sources of quality change together, I have concluded that quality changes account for between 0.45 to 0.55 percent per year of the 0.69 percent per year adjusted differential between the CPI for rent and the median housing cost. This exercise suggests, but in my opinion does not conclusively demonstrate, that the CPI for rent may have a small downward quality bias. In the next section I suggest a possible rationale for a small downward bias.

2.3 Issues about Estimating Depreciation

The Advisory Commission expresses skepticism about the depreciation adjustments made by the BLS:

> "First, we register our skepticism that the Randolph aging bias should be considered a bias in its entirety. Older units rent for less than new units for two reasons. First, they may physically deteriorate by more than is offset by repairs and maintenance. But, second, they may lose value as newer units come on the market containing amenities such as central air conditioning. Such economic obsolescence does not represent a decline in the quality of the service provided by the older apartments, but rather represents the result of the fact that the income elasticity of demand for shelter amenities is positive, and people expect higher quality in apartments and houses as the nation's per capita income increases. An exact analogy is the introduction of the jet plane…The quality of the ride on a propellor-driven DC-7 did not decline when the pure-jet DC-8 was introduced in 1958. Rather, consumers valued the ride on the jet plane so highly that the demand for flights on the DC-7 vanished. The DC-7 was scrapped prematurely, within five to ten years after the introduction of the jets. Consumers gained the entire surplus from the transition from propellor to jet planes for long-distance air travel, and the declining rents of older apartments represent a less dramatic example of the same phenomenon." (U.S. Senate, 1996, pp. 28-29)

Moving past the Commission's rather odd analogy between apartments and jet airplanes (rather than being scrapped prematurely, if anything the rate of housing demolition has dropped during the last couple of decades and renovations have increased—see Smith, Rosen, and Fallis (1988)), in my view the Commission's skepticism about Randolph's research is misplaced. Rather than overlooking technological obsolescence, separating the effects of obsolescence ("vintage effects") from age effects was the theme of Randolph's (1988a) first paper (not cited by the Advisory Commission), which developed the methodology that was subsequently applied to the CPI depreciation adjustment in Randolph (1988b).

Randolph acknowledges, as is well known in the literature, that the separation of vintage effects from age effects requires use of external identifying assumptions. In Randolph (1988a), a vintage effect trend is explicitly modeled. He then finds that if the regression conditions on both physical characteristics and neighborhood characteristics, which he considers to be correlated with the vintage effects, then the unmeasured vintage effects drop out. Randolph's model conditions on the more important observable physical characteristics such as air conditioning, and assumes that the unmeasured physical attributes, such as improved wiring or windows, are correlated with the measured attributes and/or the neighborhood characteristics. Based upon these identifying assumptions, his estimates of physical depreciation were significantly smaller than earlier estimates had been. Randolph acknowledged that the estimates may be imperfect--indeed the same could be said about any empirical estimate of depreciation--but the Advisory Commission's claim that vintage effects were ignored is simply untrue. In the absence of empirical evidence to the contrary, I see no reason to expect that the depreciation models developed by Randolph and used by the BLS overadjust for depreciation.

There are at least two reasons to think that Randolph's method may possibly understate the decline in rental value as units age. First, there is an errors in the variables problem—the age effect derives from a regression in which the age variable is reported by survey respondents. Reporting of unit age is incomplete and I suspect that some respondents may not have precise knowledge of the age of the unit. Although the direction of any errors-in-variables bias is not known with certainty, it is usually presumed to be a downward bias. Second, Randolph's method conditions on the attributes of neighborhoods in order to get an estimate limited to the physical depreciation of the unit. If the amenities provided by the neighborhood also typically decline in their value to consumers as the neighborhood ages, then Randolph's method would miss this source of depreciation. This effect, if it exists, is also likely to result in a downward bias. I emphasize that these speculations about possible downward bias are untested hypotheses, and further research would be needed to confirm or reject these hypotheses.

3. Imputation of Homeowners' Equivalent Rent in the U.S. CPI

3.1 Introduction

Owner occupied housing is a capital good owned by the consumer. Consequently the appropriate measure of the price of housing services to the owner is the user cost (Jorgenson, 1971). During the period in the late 1970s of high inflation in the U.S., it was apparent that the methods then used to calculate the homeowner index seriously overstated the inflation as measured by the user cost

(Dougherty and Van Order, 1982; Gillingham, 1983). Attempts to develop an empirical index led to the conclusion that there were empirical advantages to basing a price index on rental equivalence rather than direct measurement of the user cost. The rental equivalence approach was adopted by BLS for the CPI-U (CPI for all urban consumers) in 1983 and for the CPI-W (CPI for urban wage and clerical workers) in 1985.

Initially, the owners' equivalent rent (OER) index was calculated by simply reweighting the rent sample (Gillingham and Lane, 1982). In 1987, however, the BLS introduced a new method for calculating the CPI for homeowners' equivalent rent using data from a sample that included both owners and renters (Lane and Sommers, 1984). A representative sample of owner units was drawn, and an "implicit rent" was imputed for each owner unit.[4] Subsequently every six months each owner unit is assigned a set of matched renters and the change in the owner's implicit rent is imputed from the change in rent of the matched renters.

3.2 Implicit rent

Rental equivalence has been described as an attempt to answer the following question (U.S. Department of Labor, 1983): "How much rental income do the owners of housing units forego when they choose to occupy the units themselves instead of renting them out?" The current system divides the problem of estimating the implicit rent into two components, estimation of level and change (Lane and Sommers, 1984).

The relative change of the index is estimated each period from index relatives:

(1) $$R_{t,t-1} = \frac{\sum w_i m_{ti}}{\sum w_i m_{t-1,i}},$$

[4]The owner and the BLS field economist are each asked to estimate what the unit would rent for, and the field economist's estimate is used to establish the initial value for the implicit rent.

where $R_{t,t-1}$ is the implicit-rent relative between periods t–1 and t, the w_i are sampling weights, and m_{ti} and $m_{t-1,i}$ are the implicit rents for owner unit i in the current period and the previous period.[5] The implicit-rent relative can be rewritten as

$$(2) \qquad R_{t,t-1} = \frac{\sum w_i \left(\frac{m_{ti}}{m_{t-1,i}} \right) m_{t-1,i}}{\sum w_i m_{t-1,i}} .$$

The calculation of the current implicit rent is thus divided into two components, calculating an initial level m_{0i} and calculating relative change ($m_{ti}/m_{t-1,i}$) each subsequent period.

Implicit rent and its relative change are not observable and thus must be imputed. The formula that was used from 1987-94 was the arithmetic average ("Sauerbeck" or "Carli" index formula):

$$(3) \qquad \left(\frac{m_{ti}}{m_{t-1,i}} \right) = \hat{S}_{t,t-1} = \left(\frac{1}{n_i} \right) \sum_{j \in M_i} \frac{p_{tj}}{p_{t-1,j}} ,$$

where M_i is the set of renters j that are matched to owner i, n_i is the number of renters in that set, and p_{tj} and $p_{t-1,j}$ are the current period and previous period "pure rents" for the matched renters.[6]

The Sauerbeck formula has a well known upward bias, especially when chained as is the case here. We can illustrate the bias with the following simple example. An owner, with initial implicit rent of $1,000, is matched to two renters, each with initial pure rent of $500. The first period the rent for the first renter increases from $500 to $600, while the rent for the second is unchanged at $500. The owner's implicit-rent relative is (1.2+1) / 2 = 1.1, so the imputed implicit rent will increase to $1,000 × 1.1 = $1,100. Suppose the following period the first rent of the first renter is unchanged but the rent of the second renter now increases from $500 to $600. Again the owner's implicit rent relative

[5] The "previous period", t–1, here actually refers to the period six months previous, since rents are repriced at six-month intervals. Since January 1995, the one-month index change has been calculated as the index relative raised to the one-sixth power. From 1978 to 1994 the one-month index change was calculated from a composite estimator of the six-month relative and a one-month relative that was calculated from respondent recall of the previous month's rent. For details, see Armknecht, Moulton, and Stewart (1995).

[6] The "pure rent" deducts the value of any landlord provided utilities or furnishings, since those are not included in the implicit rent of the owner, but instead are measured in the appropriate price index for utilities or furnishings. The imputation formulas also add a depreciation adjustment factor that I will overlook here.

is (1+1.2) / 2 = 1.1, so the owner's implicit rent increases by 10 percent to 1,210. Notice that although both matched renters had total rent increases over the two periods of 20 percent, the net increase in implicit rent over the two periods was 21 percent.

The fact that (3) is upward biased has been pointed out many times by index number researchers (e.g., Fisher, 1922; Dalén, 1992, Diewert, 1995). The Sauerbeck formula (3) does not satisfy the time reversal test, nor, as our example above shows, does it satisfy the proportionality property when it is chained each period.

An alternative set of imputation formulas is defined by the generalized ratio of means. The generalized ratio of means is

(4) $$\hat{M}_{t,t-1}(\alpha) = \left(\frac{\sum p_{tj}^{\alpha}}{\sum p_{t-1,j}^{\alpha}} \right)^{1/\alpha},$$

where the summations are over the set of matched renters. These formulas satisfy the proportionality and time reversal tests of index number theory. The ratio of arithmetic means is defined as

(5) $$\hat{A}_{t,t-1} = \hat{M}_{t,t-1}(1) = \frac{\sum p_{tj}}{\sum p_{t-1,j}},$$

and the geometric mean is

(6) $$\hat{G}_{t,t-1} = \lim_{\alpha \to 0} \hat{M}_{t,t-1}(\alpha) = \prod \left(\frac{p_{tj}}{p_{t-1,j}} \right)^{1/n_i},$$

(see Abromowitz and Stegun, 1964, p. 10). I also consider another special case of $\alpha = 1/2$, which usually produces an imputation that is between $\hat{A}$ and $\hat{G}$:

(7) $$\hat{R}_{t,t-1} = \hat{M}_{t,t-1}(1/2) = \left(\frac{\sum \sqrt{p_{tj}}}{\sum \sqrt{p_{t-1,j}}} \right)^{2}.$$

The most important difference between the ratio of averages and geometric mean imputation formulas is in the relative weight given to high priced and low priced matched rental units. The geometric mean formula weights all relative changes equally, whereas the arithmetic mean weights them proportionally to the previous period's rent level. In other words, (5) can be rewritten as a weighted average of renter relatives:

$$\hat{A}_{t,t-1} = \frac{\sum p_{t-1,j}\left(\frac{p_{t,j}}{p_{t-1,j}}\right)}{\sum p_{t-1,j}}. \tag{8}$$

This shows, for example, that a 2 percent rent increase for a rental unit with $2,000 rent has the same impact on the owner's implicit rent as an 8 percent increase for a unit with $500 rent. In contrast, the geometric mean formula (6) would give all percentage changes equal weight, so a rent increase from $200 to $300 would have the same effect on implicit rent as an increase from $1,000 to $1,500. In extreme cases such as this example, the weighting implied by either of these formulas might appear undesirable, I also considered the intermediate case of (7). In practice, however, we will see that all three imputation formulas usually yield similar results.

3.3 Owner-Renter Matching

The implicit rent of each owner is calculated from a set of matched renters. How is that set determined? A fairly complex matching algorithm was introduced in 1987 with the purpose of matching owned units to rental units in the same neighborhood with similar characteristics. The algorithm was a hierarchical procedure in which the owner and renter datasets pass through up to seven stages or "partitions" to find matched renters that meet ever-weakening matching criteria. The variables used in matching are (a) segment (i.e., a small geographical area roughly corresponding to a "neighborhood"), (b) structure type (detached vs. attached), (c) value class (three classes of unit value derived from unit characteristics such as number of rooms or air conditioning), (d) rent level (three classes of neighborhood rent level), and (e) primary sampling unit (PSU, i.e., the metropolitan statistical area or non-metropolitan urban area). The best possible match would match all of these characteristics, whereas the worst would match only the PSU.

These matching criteria proved to be inadequate in several respects. First, research conducted by BLS staff consistently showed that although characteristics such as structure type or value class may be important determinants of rent *levels*, only geography tended to play a large role in explaining rent *changes* (Johnson, 1994).[7] In other words, rents within a neighborhood or a section of a PSU exhibit a tendency to move together, but after accounting for geographical location there is not a strong tendency for, e.g., detached houses to exhibit a different rate of rent inflation from attached apartments. Of the variables shown above, only segment and PSU are geographically defined.

Second, since there are typically very few sample renters within a sample segment, and the matching algorithm placed high priority on matching owners to renters within the same segment, one might expect that many owners had few matched renters. This indeed was the case. Table 2 presents the frequency distribution during the last half of 1993 of the number of renters matched to each owner. The median number of renters matched to an owner was 3, the mean was 6.9, and 25.7 percent of owners were matched to a single renter.

In the next section we conduct a simulation of the matching procedure, and in order to do so we needed to know to whether matches tended to be permanent or temporary. About 60 percent of matches in December 1993 were repeated six months earlier in June 1993, the last time those owners and renters were used in the index. Comparing matches in December 1992 to December 1993, only 47 percent of matches are retained, and only 38 percent of the matches held for all three possible matches.

3.4 Simulations

In these simulations (originally conducted during the summer of 1994) I intended to compare the statistical performance of the Sauerbeck formula (3) then in use for the implicit rent formula to three possible alternatives, (5), (6), and (7). I chose to use as my pseudo-population the historical CPI rent data because the relative performance of the estimators can be greatly affected by the dynamics of rent change. Many aspects of the dynamics are poorly understood, so I chose to use actual data

[7]Current plans for the CPI revision housing sample call for geography to play a greatly expanded role in sample stratification and selection.

rather than some assumed dynamics to better mirror the performance of the formulas under realistic conditions.

This study includes 16 pseudo-populations (4 regions by 4 monthly cycles). The data were drawn from all renters that were matched to owners during January, March, April, and May of 1989 and includes rent change through January-May 1994. Table 4 gives the population values for estimators (5), (6), and (7) for each pseudo-population.

Eight sample sizes were used in the simulations—1, 2, 3, 4, 5, 6, 8, and 10. The matching process was simulated by two sampling methods. The first is a "fixed" sample—a sample of matched renters is randomly drawn in the first period and maintained throughout the ten periods of the simulation. The second method is a 2nd order Markov model that was designed to mimic the unstable nature of the actual matching procedure. During the first period a random sample of matched renters is drawn. In the second period the renters that were matched during the first period are assigned a 60 percent probability of selection, and any additional renters are randomly selected from the remaining pseudo-population. During the third and subsequent periods the probability of selection depends on selections during the preceding two periods. If the unit had been matched during both of the previous two periods its probability of selection is 0.72. If the unit was matched during t–1 but not during t–2, it is assigned a probability of selection of 0.42. If the unit was matched during t–2 but not during t–1, it is assigned a probability of selection of 0.20. Any remaining vacancies are filled by random selection from the pseudo-population. This model emulates the declining persistence of matches as time passes.

To calculate measures of goodness of fit requires that an estimation target be defined. Since defining the estimation target was one of the issues to be addressed, I considered the population values of each of the formulas (5), (6), and (7) as possible targets. These targets are shown in Table 4.

I performed 5,000 replications (i.e., sample selections) for each of the sample size and sampling scheme for each of the 16 pseudo-populations. I report the following measures of estimator performance: long-run bias:

(9) $$\text{Bias} = \frac{1}{5{,}000}\sum_{i=1}^{5{,}000}\left(\hat{E}_{10,0,i} - T_{10,0}\right)$$

where $\hat{E}_{10,0,i}$ is one of the three long-run (or chained) estimators ($\hat{S}_{10,0,i}$, $\hat{A}_{10,0,i}$, or $\hat{G}_{10,0,i}$) and $T_{10,0}$ is one of the long-run pseudo population targets. The short-run root-mean-squared error measures the variability of the single-period imputations:

(10) $$\text{S-R RMSE} = \left[\frac{1}{50{,}000}\sum_{t=1}^{10}\sum_{i=1}^{5{,}000}\left(\hat{E}_{t,t-1,i} - T_{t,t-1}\right)^2\right]^{1/2},$$

whereas the long-run root-mean-squared error measures the variability of the chained imputations over the entire five-year period:

(11) $$\text{L-R RMSE} = \left[\frac{1}{5{,}000}\sum_{i=1}^{5{,}000}\left(\hat{E}_{10,0,i} - T_{10,0}\right)^2\right]^{1/2}$$

All of these measures of goodness of fit are then averaged across the 16 pseudo-populations.

A caveat should be mentioned in interpreting the RMSE measures. These are the RMSE for the imputation of implicit rent change for a single owner. The results will show big declines in RMSE as sample size grows, but those declines may not translate into similar declines in RMSE of the OER *index relative*. In particular, when the matching rule is changed to allow more matched renters per owner, an implication is that each renter will be matched to more owners. This creates a covariance between owners that is not shown in this table, but which prevents declines in RMSE at the level of the owner from being automatically converted into declines in RMSE of the index.

The results of the simulations are shown in Table 5. The most important finding is that the Sauerbeck formula is seriously biased upward in all of the simulations. Looking at target *A*, *n*=4 (a typical number of matched renters) and the Markov sampling method, the Sauerbeck estimator was biased upward by 3.6 percent over five years, or 0.7 percent per year.

The three alternative estimators all have fairly small bias for *n*>4. All three exhibit a bias for small samples which is very pronounced for *n*=1 (the case where all three estimators are identical). For cases where the sample size is reasonably large, however, there seemed to be little difference between the three alternatives in bias or root mean squared error. The fixed sampling method did yield lower long-run sampling error than the Markov, moving sample method.

3.5 Changes in Methods

As a result of this and other BLS research, several changes were made to the shelter indexes in 1995 and 1996. Beginning in January 1995 the ratio-of-arithmetic-means formula (5) was adopted for the implicit rent formula. This change was estimated to reduce the upward bias of the OER index by about 0.5 percent per year (Armknecht, Moulton, and Stewart, 1995). Concurrently, the index estimator for both rent and OER was changed so that the recall-based one-month rent changes would no longer be included in the index change. This change was estimated to reduce a downward bias of about 0.2 percent per year for rent and about 0.1 percent per year for OER.

Beginning in January 1996, another change was made to the matching algorithm to address the small-sample bias problem. Owners could no longer be matched to a single renter, and the matching algorithm was changed to reflect primarily geographical variables (i.e., segment and PSU). These changes tended to greatly increase the typical number of renters matched to each owner. Based upon the simulations of bias for various sample sizes and the change in the distribution of numbers of matched renters between Tables 2 and 3, my best guess is that the January 1996 change may have reduced upward small-sample bias of the OER index by about 0.1 percent per year.

Further changes are planned for the current revision housing sample starting in 1999 (Ptacek and Baskin, 1996). The BLS will drop the owner sample and return to its pre-1987 practice of estimating the OER index by applying different weights to the same sample of renters as is used in the rent index.[8] This change will eliminate the intermediate steps of matching owners to renters and imputing implicit rents to each owner. The current methods are in essence a highly complicated method of reweighting the rent changes of renters, and it is expected that eliminating the owner sample will reduce complexity and cost without compromising the quality of the OER index. In

[8] The difference in weights will primarily reflect differences in geographical location.

addition, the change in methods will reduce problems in locating sufficient numbers of renters in owner-occupied neighborhoods to support owner/renter matching.

4. Conclusions

The Advisory Commission's estimate of upward quality bias for the U.S. CPI rent index is shown to not be supported by the available data. The Commission's quality bias estimate for rent, an upward bias of +0.25 percent per year, appears to have been largely the result of an implausibly large estimate of increase in average floorspace. My corrections of their analysis lead me to conclude that the unexplained gap between the median monthly housing cost and the CPI rent index is downward, amounting to 0.15 to 0.25 percent per year, suggesting a possible downward bias of the rent index.

I also discussed research and simulations that were used to estimate the approximate magnitude and nature of an upward "formula" bias that affected the OER index from 1987 to 1994. Changes made to the CPI estimation procedures for OER in 1995 and 1996 have now largely eliminated this source of bias. Further improvements for the housing indexes are planned as part of the on-going CPI revision activities.

Table 1

						Structure type								
Year	CPI-Rent	Adjusted CPI-Rent	Median housing cost-AHS	Migration adjusted hsg cost	Median sq. feet, SF-detached & mobile[1]	Single detached (pct)	Single attached (pct)	Multi-unit 2-4 units (pct)	Multi-unit 5+ units (pct)	Mobile home (pct)	Central AC (pct)	Median rooms	2 or more baths (pct)	Mean space, new multi-unit[1]
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1993	150.3	265.3	487	296.3	1,273	25.2	7.6	22.4	41.3	3.5	34.7	4.2	16.1	1,065
1991	143.3	252.1	462	279.9	1,255	25.2	8.2	22.7	40.6	3.3	33.3	4.2	15.4	1,020
1989	132.8	233.6	424	258.0	1,272	25.6	7.2	23.7	40.4	3.1	31.9	4.2	14.5	1,000
1987	123.1	216.4	399	240.7	1,248	25.7	7.2	23.2	40.8	3.0	29.4	4.1	13.0	980
1985	111.8	199.6	364	218.8	1,245	25.9	5.9	25.5	40.1	2.6	26.9	4.1	11.7	922
1983	100.1	179.3	315	188.5	NA	27.2	4.8	26.1	39.4	2.6	21.9	4.0	10.2	942
1980	80.9	140.0	241	144.5	NA	26.8	4.3	27.1	39.2	2.6	19.0	4.0	8.4	979
1976	61.1	100.0	167	100.0	NA	32.5		27.3	37.8	2.5	16.1	4.0	6.9	948

Data sources: (1) CPI. (2) author calculations adjusting for CPI changes in methodology and including index of renter paid utilities, (3) U.S. Departments of Commerce and HUD, American Housing Survey, Table 4-13, (4) author calculations of index of changes in median housing cost holding regional distribution of renters constant at 1976 distribution, (6)-(10) percentage calculations of data from American Housing Survey, Table 4-1, (11) percentage calculations of data from American Housing Survey, Table 4-4, (12) American Housing Survey, Table 4-3, (13) percentage calculations of data from American Housing Survey, Table 4-4, (13) U.S. Department of Commerce, Characteristics of New Housing, Current Construction Reports, C-25.

[1] In square feet (1 square meter = 10.764 square feet)

Table 2
Distribution of Owner Matches by Number of Renters, July-December 1993

Number of Matched Renters	Percent	Cumulative Percent
1	25.7	25.7
2	17.3	43.0
3	11.3	54.3
4	8.6	63.0
5	6.2	69.2
6-10	13.5	82.7
11-	17.3	100.0

Table 3
Distribution of Owner Matches by Number of Renters, July-December 1996

Number of Matched Renters	Percent	Cumulative Percent
1	0.0	0.0
2	20.6	20.6
3	14.1	34.7
4	10.2	44.9
5	7.4	52.3
6-10	19.6	71.9
11-	17.3	100.0

Table 4
Population Imputation Targets for 16 Pseudo Populations, 1989-1994

Formula:	*A*	*R*	*G*
Northeast			
Panel 1	1.173	1.183	1.196
Panel 3	1.160	1.165	1.169
Panel 4	1.167	1.172	1.175
Panel 5	1.155	1.158	1.161
Midwest			
Panel 1	1.142	1.145	1.149
Panel 3	1.173	1.167	1.160
Panel 4	1.161	1.163	1.166
Panel 5	1.162	1.160	1.159
South			
Panel 1	1.159	1.163	1.167
Panel 3	1.129	1.129	1.129
Panel 4	1.150	1.148	1.146
Panel 5	1.143	1.147	1.150
West			
Panel 1	1.139	1.154	1.168
Panel 3	1.166	1.173	1.179
Panel 4	1.141	1.146	1.150
Panel 5	1.159	1.164	1.169

Table 5
Simulations of CPI Homeowner's Implicit Rent Imputation (5,000 replicates)
Averages over 16 Pseudo-Populations

	Sample Size:							
Estimator	1	2	3	4	5	6	8	10
Long-run Bias								
Target = A								
Fixed sample								
$\hat{S}$	0.022	0.030	0.033	0.033	0.035	0.035	0.036	0.037
$\hat{A}$	0.022[a]	0.010	0.006	0.005	0.004	0.003	0.003	0.002
Markov								
$\hat{S}$	0.030	0.034	0.035	0.036	0.036	0.037	0.037	0.037
$\hat{A}$	0.030[a]	0.014	0.009	0.007	0.005	0.004	0.003	0.003
Target = R								
Fixed sample								
$\hat{S}$	0.018	0.026	0.029	0.030	0.031	0.032	0.032	0.033
$\hat{R}$	0.018[a]	0.008	0.005	0.004	0.003	0.003	0.002	0.002
Markov								
$\hat{S}$	0.027	0.030	0.031	0.032	0.032	0.033	0.033	0.034
$\hat{R}$	0.027[a]	0.012	0.007	0.006	0.004	0.004	0.003	0.003
Target = G								
Fixed sample								
$\hat{S}$	0.015	0.022	0.025	0.026	0.028	0.028	0.029	0.029
$\hat{G}$	0.015[a]	0.007	0.004	0.003	0.003	0.002	0.002	0.002
Markov								
$\hat{S}$	0.023	0.026	0.027	0.028	0.028	0.029	0.029	0.030
$\hat{G}$	0.023[a]	0.011	0.006	0.005	0.004	0.003	0.002	0.002

Table 5 (continued)
Simulations of CPI Homeowner's Implicit Rent Imputation (5,000 replicates)
Averages over 16 Pseudo-Populations

	Sample Size:							
Estimator	1	2	3	4	5	6	8	10
Short-run RMSE								
Target = A								
Fixed sample								
$\hat{S}$	0.079	0.056	0.046	0.040	0.036	0.033	0.028	0.025
$\hat{A}$	0.079[a]	0.052	0.042	0.036	0.032	0.029	0.025	0.022
Markov								
$\hat{S}$	0.081	0.056	0.046	0.040	0.036	0.033	0.028	0.025
$\hat{A}$	0.081[a]	0.052	0.042	0.036	0.032	0.029	0.025	0.022
Target = R								
Fixed sample								
$\hat{S}$	0.079	0.056	0.046	0.040	0.036	0.032	0.028	0.025
$\hat{R}$	0.079[a]	0.051	0.041	0.035	0.031	0.029	0.024	0.022
Markov								
$\hat{S}$	0.081	0.056	0.046	0.040	0.036	0.033	0.028	0.025
$\hat{R}$	0.081[a]	0.051	0.041	0.035	0.031	0.029	0.024	0.022
Target = G								
Fixed sample								
$\hat{S}$	0.079	0.056	0.046	0.040	0.036	0.032	0.028	0.025
$\hat{G}$	0.079[a]	0.052	0.042	0.036	0.032	0.029	0.025	0.022
Markov								
$\hat{S}$	0.081	0.056	0.046	0.040	0.036	0.033	0.028	0.025
$\hat{G}$	0.081[a]	0.052	0.042	0.036	0.032	0.029	0.025	0.022

Table 5 (continued)
Simulations of CPI Homeowner's Implicit Rent Imputation (5,000 replicates)
Averages over 16 Pseudo-Populations

Estimator	Sample Size: 1	2	3	4	5	6	8	10
Long-run RMSE								
Target = A								
Fixed sample								
$\hat{S}$	0.208	0.157	0.134	0.118	0.108	0.100	0.089	0.082
$\hat{A}$	0.208[a]	0.130	0.103	0.088	0.078	0.071	0.061	0.054
Markov								
$\hat{S}$	0.266[a]	0.188	0.154	0.135	0.122	0.113	0.099	0.091
$\hat{A}$	0.266[a]	0.163	0.127	0.109	0.097	0.088	0.076	0.067
Target = R								
Fixed sample								
$\hat{S}$	0.208	0.156	0.133	0.116	0.107	0.098	0.087	0.080
$\hat{R}$	0.208[a]	0.130	0.103	0.088	0.078	0.071	0.061	0.054
Markov								
$\hat{S}$	0.265	0.187	0.153	0.134	0.121	0.111	0.098	0.089
$\hat{R}$	0.265[a]	0.161	0.126	0.108	0.096	0.087	0.074	0.066
Target = G								
Fixed sample								
$\hat{S}$	0.207	0.156	0.132	0.115	0.106	0.097	0.086	0.079
$\hat{G}$	0.207[a]	0.135	0.107	0.092	0.082	0.074	0.064	0.057
Markov								
$\hat{S}$	0.265	0.186	0.152	0.133	0.120	0.110	0.096	0.087
$\hat{G}$	0.265[a]	0.165	0.130	0.112	0.099	0.090	0.077	0.069

[a]The three estimators are identical when *n*=1.

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