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Measures of Poverty in Canada: Ambiguity and Conflict

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Measuring Poverty in Canada: Ambiguity and Conflict

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Abstract

New evidence is presented that suggests that Statistics Canada's Low Income Cut-Off (LICO) can misrepresent poverty trends. Statistics Canada's alternative to the LICO is the Low Income Measure (LIM), which avoids this problem by being simple. One further advantage of the LIM methodology is that it can easily be adapted to check whether measured trends are robust to reasonable changes in the underlying assumptions. Our results indicate that small changes to the adult equivalence scale, changes in the measure of household resources, and changes in the cut-off percentage of median income can each substantially alter conclusions about both the size and direction of change in the incidence of poverty. We also examine the impact of using distribution sensitive poverty measures from the Foster-Greer-Thorbecke family. The evidence points to the poverty incidence frequently being at odds with higher order poverty measures. Finally, we propose statistical tests for implementing the Foster and Shorrocks poverty ranking criteria. These tests allow the determination whether there is sufficient information for a conclusion of a change in poverty for any poverty line. The empirical results indicate that the increase in the LICO incidence between 1992 and 1996 is contradicted by declines in other poverty measures.

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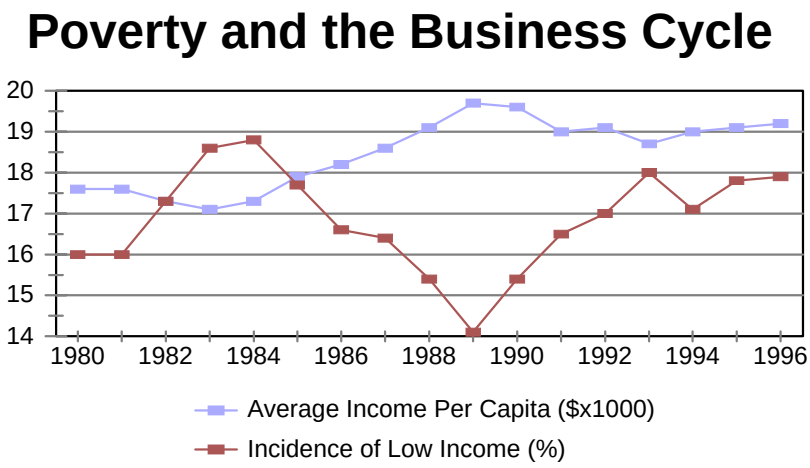
1. Introduction

The proportion of people falling below Statistics Canada's Low Income Cut-Off (LICO) is frequently used as a diagnostic test for changes in the condition of Canada's poor. The recent rise in the LICO incidence of poverty has generated some attention, appearing as it does during a time of economic expansion. (figure 1.1). Is this apparent increase a signal of a significant social change?

Figure 1.1



The simple answer is that these increases may signal nothing. Everything depends upon the whether the LICO incidence of low income is a good indicator of the changing economic status of Canada's poor. In the next section of the paper, some of the more important shortcomings of the LICO methodology are reviewed, and new empirical evidence of a serious flaw in the econometric methodology is presented. Following this, simpler, more robust poverty indicators based on Statistics Canada's Low Income Measure (LIM) methodology are proposed and estimated.

Even with more robust indicators of poverty status, the incidence measure of poverty is a crude social index for determining the size of the poverty problem. In section 3, the shortcomings of the incidence measure are reviewed, and the use of several alternative poverty indices that have received little empirical attention are recommended. Estimates of these higher order poverty indices follow, with special attention to the questions of whether poverty has been rising over the mid 1990's.

Even with the use of more robust poverty lines and poverty indices, apparent trends in sample

data can be rejected on two counts. First of all, the lines reflect the assumptions of the investigator, and there is a legitimate concern that alternative assumptions might generate a different result. Sensitivity analysis in the robust LIM and LCM approach do go someway in answering these questions, but the whole range of assumptions on the poverty line or on the poverty index used cannot be tabulated. Second, the poverty indices have been calculated from sample data, and no attempts are made in the sections 2 and 3 to formally test whether the data provides sufficient evidence of change. Without a statistical test it is legitimate to wonder whether the indicated trends are simply artefacts of a limited sample.

To address these concerns, a poverty ranking criteria is proposed in section 4, and non-parametric statistical tests are developed. By applying these tests to the data, it is possible to statistically test whether the the indicated changes in sections 2 and 3 are consistent with the choice of any reasonable absolute poverty line. Following a discussion of the results, conclusions are drawn in section 5.

All poverty indices and poverty dominance tests in this study are estimated on the micro-data set from Statistics Canada's Family Expenditure Survey for the years 1978, 1982, 1986, 1992, and 1996. These years were chosen because they are the only available surveys that examine both income and expenditures on a national scale. While there are other micro-datasets, the FAMEX is unique in allowing investigation of both income and consumption patterns.

2. Identification of the Poor

> "By necessaries I understand, not only the commodities which are indispenably necessary for the support of life, but whatever the custom of the country renders it indecent for creditable people, even the lowest order, to be without". (Smith, 1776, 821-2)

Poverty is difficult to diagnose because perceptions of its symptoms vary. The consequence is a an abundance of methodologies for identifying the poor. Despite the choice, most attention in Canada is paid to Statistics Canada's Low Income Cut-Off (LICO) as the threshold income that divides the poor from the non-poor. The source of this authority is surprising since Statistics Canada is always careful to warn against this use. In what follows, a new reason for heeding this warning will be presented, and some simpler, more robust standards will be proposed.

The LICO lines have perhaps received their authority as poverty lines because the Economic Council of Canada (1968) was an early user of the LICO for this purpose, and because the lack of any alternative from Statistics Canada, at least until recently. Beyond the prestige of these two agencies, the appeal of the LICO may also rest on its use of econometric techniques in the determination of an appropriate line and adult equivalence scale. This can lead to an impression that the LICO is less arbitrary and more precise than the numerous alternatives. Unfortunately, appearances are deceiving. Hidden within the estimation procedure in which the LICO line is based (and periodically re-based) are a number of arbitrary assumptions that strongly affect the outcome of the estimation process. Furthermore, the estimation process is itself deeply flawed, leading to poverty lines that have the potential to mis-inform public debate. Each problem will be dealt with in turn.

The LICO methodology begins by determining the average proportion of income spend on food shelter and clothing. Then an arbitrary 20% is added to this proportion to determine the cut-off percentage of income devoted to necessities that reflect straightened circumstances. Finally, a simple Engel curve is estimated using household expenditure data,

$$\log(E) = a + b \cdot \log(Y) + \sum c_j D_j + u$$

where E = total expenditure on food shelter and clothing

Y = family income, before tax

D_j = dummy variables indicating family size, and location characteristics of residence

u = error term.

The estimated relationship between income and expenditures on necessities is then used to estimate the income that, on average, a household would need to just afford the cut-off share of necessities.

No doubt part of the appeal of this approach lies in the ease with which the LICO can be re-estimated whenever new family consumption survey data becomes available (about every four years). For those that prefer a relative poverty standard, this ability to automatically adjust the poverty line to changing consumption norms through time is a compelling reason for adopting the LICO as a poverty standard. While there are numerous other relative poverty standards that could be used, the LICO methodology is unique (in Canada) in using family behaviour to adjust for differences in consumption caused by family structure or family location. Wolfson and Evans (1989,27-28) found that further refinements of the basic estimation equation failed to improve the statistical fit of the estimated Engel curves. The apparent implication is that the LICO methodology seems to be a statistically robust technique for identifying an intuitive household income threshold. Families below the LICO must spend at least 20% more of the household budget on necessities than the community average.

Unfortunately, these apparent advantages do not hold up well under closer scrutiny. First of all the community average expenditure on necessities is simply total expenditures on necessities by all families divided by total income of all families. By calculating it this way, richer families count for more than poorer families, and the average necessities share is biassed significantly downwards, which in turn significantly biases the LICO and the poverty incidence up (Wolfson and Evans (1989), 18). The 20% parameter is arbitrary and does not reflect any precise notion of deprivation. Altering this assumption by even a few percentage points can change the estimated incidence of poverty by a substantial amount (Love, 1984). Finally, the families identified as being in low income may or may not spend a 20% greater share of their income than the average on necessities. As Wolfson and Evans (1989, 16-17) point out, the Engel curve does not fit the data very well at all.

There are many who are identified as poor who spend a significantly smaller proportion of income on necessities than the cut-off, and many non-poor who spend a significantly higher proportion on necessities than the cut-off proportion. Because of this poor fit, the LICO incidence has the potential to misinform public debate on poverty.

It might be argued that the LICO is a good compromise. After all there is no getting around the arbitrariness of setting a relative community standard, and the poor fit of the Engel curve is simply an unavoidable statistical fact. At least the methodology relies on family behaviour to estimated the income cut-offs and adjust these cut-offs by family structure and location of residence. Unfortunately, even these advantages are illusory.

While it is true that the existing specification is not improved by adding further variables this does not mean that the specification is a good one. The dummy variables for household location that are used to adjust the LICO for region and size of community do not perform very well (Wolfson and Evans, 73-77), and they conclude that the adjustments for geographical variation are likely to be too large (31). The implication is that the LICO standard could be simplified and improved by removing the geographical variability.

Beyond the question of what should or should not be in the estimating equations is the question of whether the log-linear specification is suitable to the task of estimating the Engel curves. The original choice no doubt comes from the results of Prais and Houthakker's (1955) classic study. One problem with this form is that it fails the theoretical test of adding up (Deaton and Muellbauer, 1980, 17). A second problem is that the specification assumes a constant income elasticity of demand. Alternatives that avoid these difficulties by having stronger choice theoretic foundations do exist, though they have not been much explored in Canada.

Detailed income elasticities for food shelter and clothing for Canada are hard to come by[1] and none have been calculated for specific groups defined by their location in the income distribution. They are usually obtained via the Engel curve component of the corresponding demand equation.

[1] Nicol (1993) provides estimates for the food category for 1978 and 1986 for specific family types in the process of comparing parametric and non-parametric estimation techniques. Anderson and Blundell (1983) provide aggregate food and clothing estimates in the context of modelling dynamic demand equations. All estimates are substantially below one.

One demand system that avoids the adding up problem of the LICO methodology is the Almost Ideal Demand System (AIDS), which can be altered in the same manner as the LICO methodology to examine the impact of demographic differences on consumption patterns. The conventional formulation of the Engel curve expresses the share of total expenditure made by the i'th household on the j'th commodity group (w_{ijt}) as a linear function of the natural logarithm of its total expenditure ($\alpha_j+\beta_j \ln(y_{it})$). The LICO methodology focusses attention on the Engel curve for necessities (food, shelter and clothing), and adds variables for demographic impacts on consumption. To adapt the Almost Ideal Demand System to do the same thing, begin with three demand equations for food, shelter, and clothing and add demographic variables. The result is the following equation for j = 1,2,3:

$$w_{ijt} = \alpha_j(D_{it}) + (\beta_j + \beta_{jc} I(nc_{it}>0))\left(\ln\left(\frac{y_{it}}{(na_{it}+nc_{it})p_t}\right)\right) + (\gamma_j + \gamma_{jc} I(nc_{it}>0))\ln(na_i + nc_i) + \sum_{j=1}^{3} \delta_j lnp_{jt} + e_{ijt}$$

where D_i are dummy variables reflecting household makeup, na_i and nc_i respectively correspond to the numbers of adults and children in the i'th household in period t, I( ) is an indicator function, p is an overall price index and p_{jt} ,j=1,.,3 correspond respectively to price indices for food, shelter and clothing in period t. Under this extension to the Almost Ideal Demand System, γ=0 implies a household scale elasticity of 1 (this reflects no economy of scale in consumption see Lanjouw and Ravallion (1995) for details)[2]. The Income elasticity implied by this formulation is:

$$e_{ij} = \frac{\beta_j + \beta_{jc} I(nc>0)}{w_{ij}} + 1$$

with an approximate standard error of the elasticity estimate is given by:

[2] Note this is not the true adult equivalence scale (defined as the scaling factor by which the cost functions of households of differing composition should be equated at a given utility level for a given price level) which cannot be identified by demand data alone (see Blundell and Lewbel (1991)).

$$se(e_{ij}) = \frac{se(\beta_j^*)}{w_{ij}}$$

where se(β_j^*) is the standard error of β_j^*, the estimator of $\beta_j + \beta_{jc} I(nc>0)$.

Using the 1982 to 1992 FAMEX micro-data sets[3], approximately 1000 observations were randomly selected from each survey after eliminating all units who recorded less that $100 on each of the expenditure categories and less than $500 on total consumption or receiving less than $500 in income. The estimation results are presented in Table 2.1

Table 2.1 Expenditure shares and income elasticities 1982-1992

	Food	**Shelter**	**Clothing**
means	0.2028	0.2553	0.0759
medians	0.1933	0.2395	0.0681
minimum	0.0123	0.0086	0.0027
maximum	0.5719	0.8881	0.4256
elasticity (nc) ("t's"(h_0:e=1))	0.5985 (49.3656)	0.7338 (50.1896)	1.1843 (56.0867)
elasticity ("t's"(h_0:e=1))	0.5928 (155.9894)	0.7898 (136.8969)	1.2195 (492.0113)

[3]Comparable information on family structure for the 1978 sample was not available.

Income Elasticities ("t's"(h_0:e=1)) for selected quantiles (no children)

0 < .125	0.7232 (2.8010)	1.0323 (0.2025)	0.8545 (0.5341)
.125 < .25	0.9806 (0.0611)	0.6501 (0.7690)	1.8579 (1.1860)
.25 < .5	0.5650 (7.8121)	0.6709 (4.6928)	1.2931 (2.7441)
.5 < .75	0.3477 (6.3771)	0.5687 (3.6648)	1.0551 (0.3410)
.75 < 1.0	0.6414 (8.2658)	0.6984 (6.9676)	1.0301 (0.5266)

Income Elasticities ("t's"(h_0:e=1)) for selected quantiles (children)

	Food	**Shelter**	**Clothing**
0 < .125	0.8632 (2.1056)	0.8144 (1.7685)	0.9255 (0.4158)
.125 < .25	1.0641 (0.1918)	0.4037 (1.2418)	1.6311 (0.8268)
.25 < .5	0.6112 (6.8509)	0.8019 (2.7709)	1.1497 (1.3750)
.5 < .75	0.3264 (5.2136)	0.6843 (2.1237)	0.9745 (0.1252)
.75 < 1.0	0.6744 (2.2625)	0.9061 (0.6540)	1.0998 (0.5269)

The overall elasticities are reported as well as the elasticities of a set of quantile regressions. $\chi^2(44)$ hypothesis tests (P values) of the restrictions implied by the overall model over the quantile regressions for food, shelter and clothing are respectively 214.0 (3.41e-24), 176.3 (9.52e-18) and 132.6 (8.07e-11) suggesting substantial and significant variability of Engel curve structures over the income distribution[4]. The results indicate income elasticities of around 1 for all three consumption

[4] Regression coefficients (not reported here for space reasons) indicate household size elasticities differed across commodity groups (approximately .6 for food, 0 for shelter and 1 for clothing).

categories in the low income groups (i.e. in the lowest quartile) and significantly less than one elsewhere for food and shelter. The presence of children appears to have little effect on these elasticities, the conclusion that can be drawn is that increasing the income of the poor might well cause their consumption of necessities to rise as a share of their income. The implication is that the Engel curve estimated by the LICO methodology seriously misrepresents the true relationship between income and the expenditures on necessities by the poor. In other words the LICO income lines do not provide a reasonable estimate of the income level below which expenditures on income will be at least 20% higher than the community norm.

One final bit of criticism is that the LICO methodology was a failed attempt to develop a relative measure of poverty. The fact that it is only re-based every four years, leads to the unusual result of a predictable increase in poverty every time that there is a new estimated base. If one holds to a pure relative measure, the poverty rate should only be calculated in years in which a FAMEX survey is done. The practice that has evolved is to use a single LICO poverty base to track poverty over time, resulting in an absolute standard.

So what poverty standards should be employed? There are many alternatives to the LICO that have been proposed and used in identifying the poor in Canada. Determining the best among alternatives will have to wait until a clear consensus emerges over exactly what poverty is. As things stand now, there is not even a consensus about whether poverty is best conceived under a welfarist or entitlement framework, and whether it is an absolute or relative condition.

The attempt to provide a relative poverty measure in the LICO methodology therefore cannot be easily swept aside.[5] Even Amartya Sen's influential voice in favour of an absolutist standard admits that in observable indicators of household economic power, poverty might best be conceived as a relative concept.[6] Statistics Canada's recent proposal for a Low Income Measure (LIM) as an

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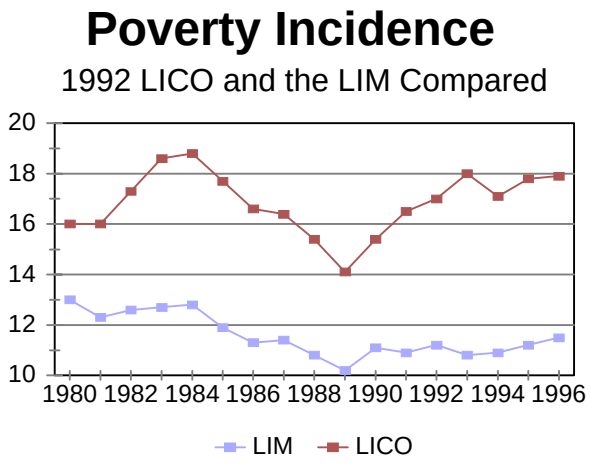
[5]Sarlo (1996) has been perhaps the most prominent among Canadian researchers in arguing against the relative notion of poverty implicit in the LICO. His alternative of defining an income sufficient to afford a minimum standard level of necessities has a pedigree stretching back to Rowntree (1901). The problem is that Sarlo's absolute standard has an obvious relative core that is apparent if the suggested budget is examined.

[6]"Poverty is an absolute notion in the space of capabilities but very often it will take the relative form in the space of commodities or characteristics." (Sen, 1983, 161)

alternate measure of relative deprivation succeeds where the LICO fails. It is an obviously arbitrary standard, with obviously arbitrary adult equivalent scales, having the added cachet of being similar to the proposed OECD methodology (OECD, 1982). Its simplicity makes it difficult to be misinterpreted - providing a level of reliability that the LICO cannot provide. Finally, the ability to make annual adjustments to the poverty standard makes the LIM a true relative measure of poverty.

Under the LIM, poverty trends over the last fifteen years look similar in terms of the trends in poverty by age, and in the pattern over the business cycle. The main difference seems to be that the LICO indicates that poverty in 1996 was worse than it was in 1980, while the LIM suggests that there was some improvement (see figure 2.1). Both show the same counter-cyclical rise between 1994 and 1996.

Figure 2.1

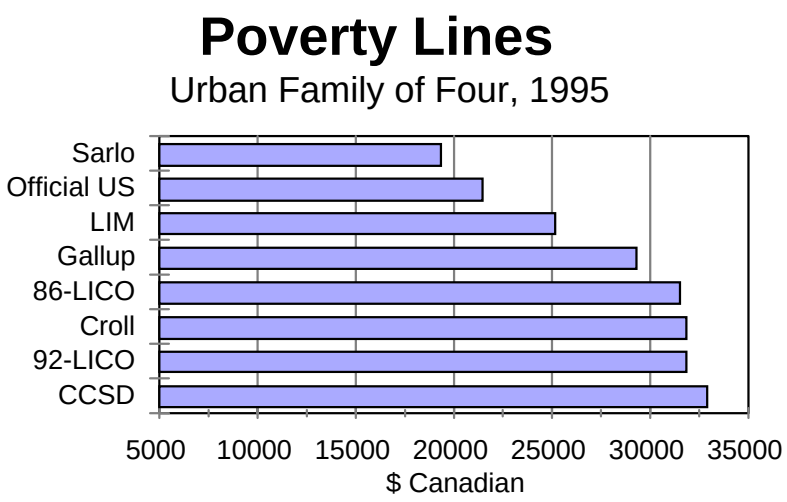


Despite the attractions of the LIM as a relative poverty measure, there are a number of objections that might be advanced against it. The first is the use of an arbitrary adult equivalence scale that is dramatically different from the estimated adult equivalent scales that come out of the LICO methodology.

Numerous procedures for estimating adult equivalence scales with a stronger choice theoretic basis than found in the LICO methodology have been developed.[7] An empirical problem identified in this research is that theoretically desirable equivalence scales are probably not identifiable

[7]The attempts to provide a stronger choice theoretic basis for adult equivalence scales can be traced to Barton (1964) who derived adult equivalence scales using a direct utility function. Muellbauer (1974) used indirect methods to achieve the same end.

(Blundell and Lewbell,1991). Even if they were, they would rest on an assumption of comparable utility. This gets the researcher back to the problem of deciding what is a reasonable adult equivalence scale, and based on practice, there appears to be a wide variety of opinion concerning what is reasonable.



Buhmann *et al.* (1989) examined 34 different equivalence scales and discovered that each could be closely approximated by estimating $Y = M / d^{\varepsilon}$, where Y is equivalent income, M is money income, d is family size, and ε is an elasticity of family size. Estimation results indicated that the value for ε depends on the methodology used to estimate the equivalence scale. Expert programme scales used in welfare programs typically have a family size elasticity of about 0.55, while equivalence scales estimated from observed consumption behaviour come in at about 0.36. The 1986 LICO lines had an adult equivalence scale that had an elasticity of 0.56. Based upon this finding, we decided to use a household elasticity of 0.5 to form our base adult equivalence scale, and use elasticities of 0.4 and 0.6 to check for the robustness of the results.

A second objection to the LIM methodology is the use of 50% of median income as the poverty line. Numerous alternatives have been proposed, with the LIM being substantially below many popular lines, though significantly above the official US poverty line (figure 2.2).

Figure 2.2[8]

[8]These poverty lines are taken from the National Council of Welfare's (1997) Poverty Profile, 1995 and from the US Bureau of Labour Statistics.

It is straightforward to extend the LIM methodology to examine poverty trends under a 40% and 60% of median income assumption. The 40% of median income assumption provides a poverty line below the US official poverty line, while the 60% of median income assumption provides a poverty line above the Gallup survey line, and very close to the LICO lines. Doing this allows trends to be examined under different, intuitive assumptions of what constitutes deprivation.

The final problem with the LIM, and for that matter all the other poverty and low income lines in use, is their use of before tax income. Statistics Canada has recently developed after tax LICO and LIM thresholds and low income statistics (cat.13-592). While this is an improvement over before tax income, there are many reasons for questioning the validity of any income measure in the identification of the adequacy of household resources. If concern is with the level of resources available to the household, the appropriate measure would be wealth, or perhaps an annuity value of wealth (permanent income) as proposed by Weisbrod and Hansen (1968). The difficulty of developing reliable wealth estimates has lead to the use of consumption expenditures as a more reliable indicator of permanent income. Of course this is not without its difficulties. One problem is that liquidity constraints facing many poor families weakens the connection between wealth and spending. A second problem is that poor families have a lower level of ownership of consumer durables, and so the relationship between consumption expenditures and actual consumption is different than for more prosperous families (Kay *et al.*, 1984, 170). Despite these problems, consumption expenditures remain an attractive method for measuring household living standards.

The point of the LICO measure though was to measure the adequacy of income for affording necessities. As Atkinson (1989, 10) points out, this concern for the tolerable minimum level of necessities does not fit within the usual welfare framework that economists are familiar with, residing somewhere closer to the 'specific egalitarianism' identified by Tobin (1970). Certainly

concern about child poverty is focussed on whether the necessities are affordable. If concern about poverty is really a concern about the adequacy of necessities consumption, then neither income nor total consumption expenditures would be appropriate to the task of measuring deprivation. Both are often weakly correlated with the consumption of necessities. The best way of getting a picture of the adequacy of necessities consumption is to measure necessities expenditures directly.

This raises the question of whether income is at all useful in measuring the adequacy of household resources. Atkinson (1989, 13) suggests that where poverty is concerned with the right to a certain level of participation in society (Adam Smith's notion), a minimum level of income might well be more appropriate than consumption as an indicator of whether individual rights are being fully met. Consumption might be more appropriate if concern rests primarily on the level of welfare attained by the poor family. The proviso of course is that if income is used as an indicator of individual rights, it should be income after taxes and transfers.[9]

The bottom line is that income (after taxes and transfers), consumption expenditures, and expenditures on necessities can each claim some authority in measuring an aspect of the adequacy of household resources, and each should be considered when examining the state of poverty in Canada. Each can be easily integrated in the LIM methodology to supplement the picture provided by income.

Our proposal is for a Robust Low Income Measure (RLIM), a Robust Low Consumption Measure (RLCM), and a Robust Low Necessities Measure (RLNM). The construction of this set would use income after tax, total consumption expenditures, and total necessities expenditures (food, shelter, clothing) as the respective indicators of household resources. Conversion of household resources would be based on the Buhmann *et al.* (1989) conversion using household size elasticities of 0.4, 0.5, 0.6. After the individual resources available are associated with each individual in the sample, the poverty line can be determined as 40%, 50%, and 60% of the median of the adjusted (individualized) sample.

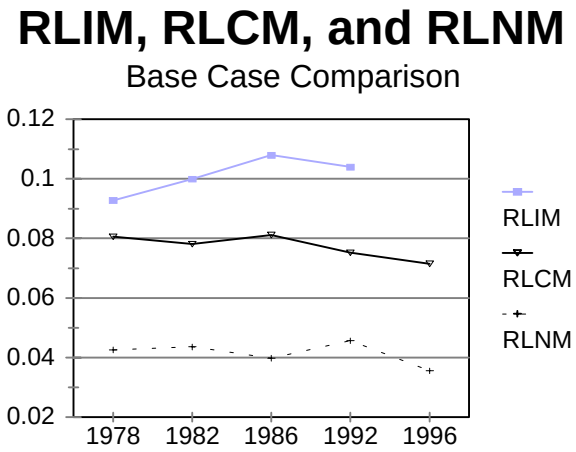
While the multiplicity of standards (27 in all) has its drawbacks, the absence of consensus on what would constitute an unequivocal poverty test leaves researchers with the task of constructing

[9]Recent income after tax LICO and LIM statistics have been published by Statistics Canada (cat. 13-592).

a set of symptoms that fit the poverty diagnosis. Some of the complexity can be reduced by agreeing on a base case and doing sensitivity analysis. For instance, a household size elasticity of 0.5 might be an appropriate base case for household size elasticity, with sensitivity analysis done for elasticities of 0.4 and 0.6. Similarly, 50% of the median might be considered the standard cut-off, with sensitivity done for 40% and 60%. However the complexity problem is dealt with, areas of real ambiguity in poverty measurement should be made clear to policy makers and the public.

Given these relative standards, the trends in poverty incidence indicate that under the base case assumptions (the household size elasticity = 0.5, while the cut-off = 50% of the median), the

Figure 2.3



trend in poverty seems to depend on the measure of household resources. Using income as a measure, poverty seems to have risen above the levels seen in the late 70's. Using consumption expenditures or necessities expenditures as a measure of household resources, the poverty trend indicates a decline in poverty. The impact of varying the adult equivalence scale assumptions used in constructing the RLIM measure are presented in table 2.2. The important point to note that changes in the assumption of the adult equivalence scale and the cut-

off can alter the measured direction of change in poverty.

Table 2.2 Sensitivity of Poverty Incidence to Changes in Adult Equivalence Scale

	RLIM		RLCM		RLNM	
Year\Elasticity	E=0.4	E=0.6	E=0.4	E=0.6	E=0.4	E=0.6
1978	0.0995	0.0889	0.0850	0.0751	0.0494	0.0386
1982	0.1055	0.0962	0.0831	0.0722	0.0491	0.0407
1986	0.1114	0.1026	0.0857	0.0763	0.0431	0.0398
1992	0.1124	0.0996	0.0826	0.0718	0.0479	0.0422
1996			0.0808	0.0666	0.0400	0.0344

Sensitivity of Poverty Incidence to Changes in Cut-Off

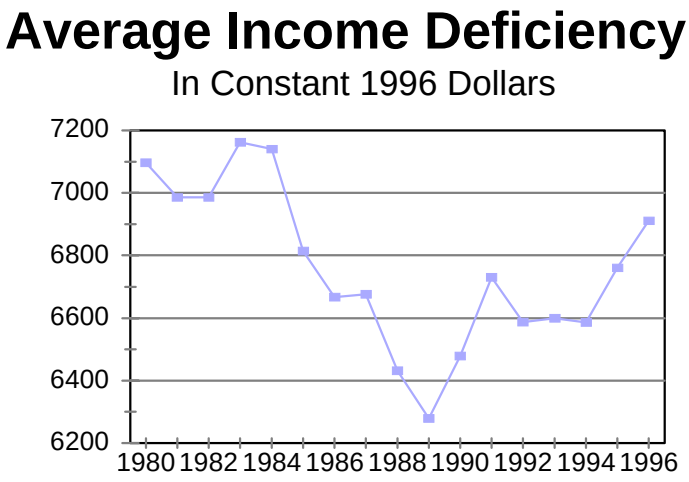
	RLIM		RLCM		RLNM	
Year\Cut-Off	40%	60%	40%	60%	40%	60%
1978	0.0437	0.1641	0.0304	0.1412	0.0129	0.0993
1982	0.0523	0.1638	0.0295	0.1396	0.0141	0.1039
1986	0.0543	0.1743	0.0306	0.1498	0.0115	0.0963
1992	0.0506	0.1701	0.0253	0.1413	0.0138	0.0994
1996			0.0241	0.1365	0.0108	0.0934

Between 1986 and 1992 poverty went up under the RLIM if the adult equivalence scale used had an elasticity of around 0.4. If the elasticity was 0.6, however, the evidence indicates that poverty declined. Between 1978 and 1982, the RLIM indicates that the poverty incidence increased if a 40% cut-off was used, while the poverty incidence seemed to go down under a 60% cut-off. The conclusion that should be drawn from this is that the choice of household resource, adult equivalence scale, and relative threshold values for the cut-off can each affect more than the level of poverty. It can affect even the measured direction of change.

3. Measuring Poverty

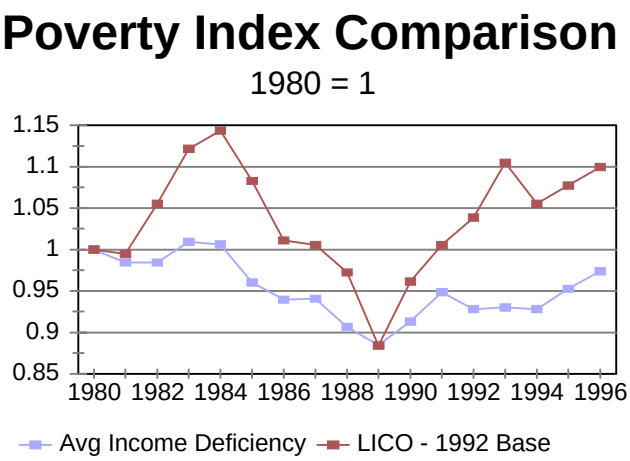
Beyond the controversies surrounding identification of the poor is the problem of aggregation. While poverty incidence receives the most attention among the policy community, it arbitrarily assumes that every poor person contributes 1/n to the poverty problem, where n is the size of the population. The size of the poverty problem is then simply q/n, where q is the number of poor individuals. As is well known, this fails to account for changes in the depth of poverty. Because of this known deficiency in the incidence measure of poverty, Statistics Canada and other groups that track Canadian poverty also publish the average poverty gap. As is evident in figure 3.1, the poverty gap declined over the 1980's, and has begun a slow rise in the 1990's.

Figure 3.1



As is illustrated in figure 3.2, the path of the average income deficiency has followed a similar trend to the LICO poverty incidence. There is one important difference. The LICO indicates that poverty was a more serious problem in 1996 than in 1980, while the average income deficiency indicates the opposite. The dilemma that emerges is how to interpret the increase in the incidence of poverty given declines in the average poverty gap.

Figure 3.2



The most commonly suggested solution to this dilemma is to develop a measure that weights people further from the poverty line more heavily than people closer to the poverty line. The range of reasonable alternatives that have been proposed is truly remarkable (Zheng, 1997). Even if attention is restricted to poverty measures that meet every property (axiom) that has been identified as desirable in a poverty measure, no ideal poverty measure or index emerges.

This would not be a problem, except that the choice made might determine the outcome. Take, for example, the Foster,Greer,Thorbecke (1984) class of poverty indices,

$$FGT = \frac{1}{n}\sum_{i=1}^{q}\left(1-\frac{x_i}{z}\right)^{\alpha}$$

where z is the poverty line, x_i is the income of poor family i, n is the total population, and α is the scaling factor for the family of poverty indices. Many members of members of this class satisfy all the core axioms identified by Zheng.

When α is 0, the poverty index is simply the poverty incidence. For α=1, the poverty index is the product of the poverty incidence and the income gap ratio (average income gap expressed as a percentage of the poverty line). As α rises, the weighting of families rises for those far from the poverty line, and falls for those close to the poverty line. For $\alpha \in (1,2]$, the poverty index satisfies all the major axioms usually expected of a poverty index, except for the weak transfer axiom. For α = 2, the measure can also be decomposed into the product of poverty incidence, and a combination

of the income gap ratio and the coefficient of variation of incomes among the poor. For $\alpha > 2$, the weak transfer axiom also holds. For most practical purposes then, calculating the FGT poverty index for $\alpha = 2$ and $\alpha = 3$ should provide good measures of poverty, with the $\alpha = 3$ having a more strongly biassed weighting scheme.

Estimates of the FGT index for $\alpha = 0,1,2$, and 3 were made under the base-line assumptions.[10] The estimates were normalized to 1 for 1978 (for ease of comparison). The estimates based on after tax income are graphed in figure 3.3 as FGT0, FGT1, FGT2, and FGT3 respectively.

Figure 3.3

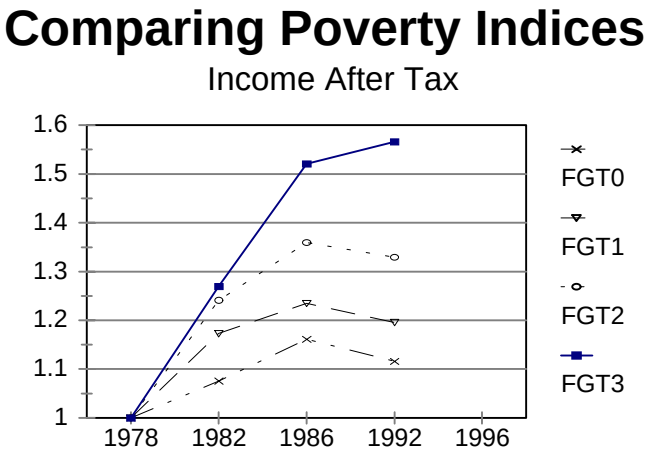
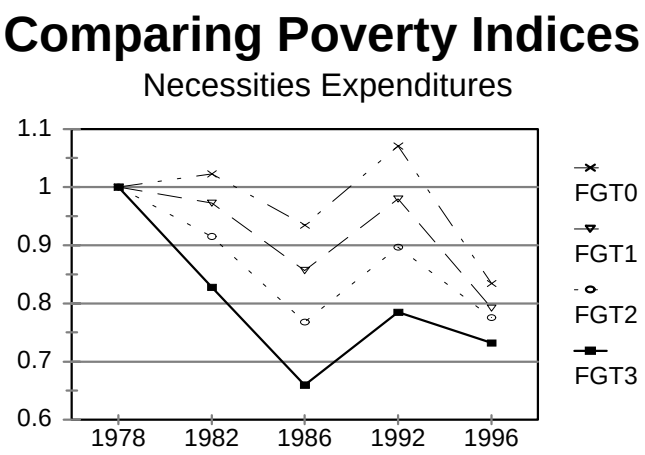


Figure 3.3 shows the highest order poverty index (FGT3) to be rising between 1986 and 1992 while all other poverty indices indicate declining poverty. When consumption expenditures are used as an indicator of household resources, the poverty incidence falls between 1992 and 1996, while all the higher order poverty indices indicate that poverty is rising (figure 3.4).

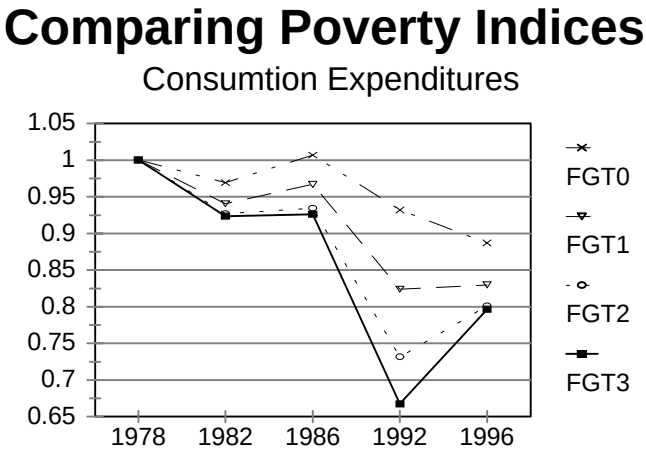
[10]Under the baseline assumptions the equivalence scale is based on a household size elasticity of 0.5 and a cut-off of 50% of the median.

Figure 3.4



Between 1978 and 1982, the FGT0 calculations on necessities expenditures also showed a different trend than the higher order poverty indices (figure 3.5). In each cases, the choice of distribution sensitive measure of poverty can alter perceptions about the trends in poverty.

Figure 3.5



Disagreement between indices of the
same family would seem to be a fairly common occurrence.

Given this possibility it would seem wiser to choose a higher level index of poverty if a single poverty index is to be reported. The common practice of focussing attention on the poverty incidence has little to recommend it.

It should be noted that simply using a higher order poverty index does not eliminate ambiguity about the trends in poverty. As the above graphs show, the choice of household resource measure largely determines the perception about the direction of change. No alternative aggregation strategy will eliminate this basic ambiguity.

4. Poverty Rankings

The discovery that poverty may be shown to go up or down depending on the choice of the poverty line, household resource indicator, adult equivalence scale, and aggregation strategy (poverty index) is perhaps not surprising. This raises the question of whether there are conditions under which poverty can be said to have unambiguously increased or decreased. Beginning with Atkinson (1987), a growing literature on poverty dominance rankings have sought such conditions. The Foster and Shorrocks (1988) approach identified a direct correspondence between α-degree stochastic dominance rankings and the poverty orderings from any α-degree FGT poverty index.[11] These orderings are unambiguous in the sense that if stochastic dominance can be shown between two distributions, then the dominating distribution has lower poverty no matter which poverty line is chosen.

Let Y be the range space from two income distributions A and B described by distribution functions $f_A(y)$ and $f_B(y)$ respectively and for notational convenience define:

$$F^{i}(y) = \int_0^y F^{i-1}(z)dz$$

where $F^0(y) = f(y)$. Order "i"stochastic dominance of f_A over f_B (denoted $f_A \succeq_i f_B$) is defined as:

$$F^i_A(y) \leq F^i_B(y) \; \forall \; y \in Y \text{ and } F^i_A(y) \neq F^i_B(y) \text{ for some } y \in Y \qquad [1]$$

The demonstration of First Order Stochastic Dominance is a sufficient condition to conclude that there exists a social preference of f_A over f_B for *any* monotonic utilitarian social welfare function. Similarly, Second Order Stochastic Dominance corresponds to a social preference of f_A over f_B for *any* utilitarian social welfare function that is increasing in mean preserving progressive transfers. Finally, Third Order Stochastic Dominance implies a social preference of f_A over f_B for *any* utilitarian social welfare function that is increasing in mean preserving progressive transfers and is increasing

[11]Jenkins and Lambert (1996) have developed a similar set of ranking criteria.

at a greater rate for such transfers at lower income levels. Furthermore, i'th order dominance implies j'th order dominance for all j > i. This relationship admits a wider range of characterisations of orderings than is possible using Generalised Lorenz curves[12]. Given suitable empirical approximations to these theoretical concepts, comparisons can be made between income distributions of the same group over time (providing information on the progress of social welfare within a group) and between groups at a particular point in time (providing information on the comparative welfare of two groups). It is also important to note that this dominance relationship is transitive in that if $f_A(y) \succeq_i f_B(y) \succeq_i f_C(y)$ then $f_A(y)$ correspondingly dominates $f_C(y)$.

Tests for condition [1] were developed in Anderson (1996), based upon a common partitioning of the range space of the variable Y and forming a discrete empirical analogue of [1]. The partition points have a well defined asymptotic distribution, providing a means for testing for stochastic dominance and the implied welfare ranking. The tests for stochastic dominance can easily be modified to examine the plight of the poor over time in the context of some given poverty line which shall be denoted by Y^p. Following Atkinson (1987) this simply involves contemplating the inequalities:

$$F^i_A(y) \leq F^i_B(y) \; \forall \; y < Y^p \text{ and } F^i_A(y) \neq F^i_B(y) \text{ for some } y < Y^p \qquad [2]$$

The tests employed for [1] are simply applied to [2] over the range $0,Y^p$ rather than over the whole range of Y.[13] First order dominance of f_A over f_B over the range $0,Y^p$ corresponds to lower poverty incidence for all poverty lines below Y^p. Second order and third order dominance imply that there

[12]Lorenz curves are a popular means of comparison (see Lambert (1993) for a discussion). Statistical comparison was facilitated by the development of asymptotic distributions of Lorenz curve ordinates (Beach and Davidson (1983)) and joint confidence bands for such ordinates (Beach and Richmond (1985)). Concerns regarding Lorenz curves (Sen (1973)) lead to the development of the Generalised Lorenz curve (Shorrocks (1983)) (the asymptotic distribution of its ordinates had in fact been derived implicitly by Beach and Davidson (1983)). However the range of statements that can be made using Generalised Lorenz curve comparisons remains limited and is essentially equivalent to confining analysis to second order dominance comparisons.

[13]Details of the statistics are reported in Anderson (1996).

is lower poverty using any poverty line below Y^p if calculated by the FGT1 and FGT2 indices respectively. An interesting byproduct of this procedure is that the last element of the first, second and third order dominance tests can used as a test for changes in the FGT0, FGT1, and FGT2 indices using the Y^p poverty standard.[14]

Estimates of the poverty dominance test statistics (first, second and third order) were made using three indicators constructed from the FAMEX micro-data set. The baseline assumptions for the construction of these indicators was a household elasticity of 0.5 and Y^p equal to 60% of the 1986 median for income after tax, consumption expenditures, and necessities expenditures.[15] The final elements for the first, second and third order poverty dominance test statistics using Income After Tax (Y), Consumption Expenditures (C), and Necessities Expenditures (N) are reported with their asymptotic standard errors in Table 4.1.

Table 4.1

Year (A vs B)		$FGT0_A$ - $FGT0_B$	$FGT1_A$ - $FGT1_B$	$FGT2_A$ - $FGT2_B$
1978 vs 1982 :	Y	0.0115 (0.00308)*	-0.000330 (0.00207)	-0.00344 (0.00137)*
	C	-0.0126 (0.00297)*	-0.00517 (0.00185)*	-0.00279 (0.00123)*
	N	-0.0140 (0.00244)*	-0.00577 (0.00131)*	-0.00288 (0.000920)*
1982 vs 1986 :	Y	-0.00119 (0.00308)*	-0.00629 (0.00211)*	-0.00265 (0.00139)
	C	0.0112 (0.00298)*	0.00894 (0.00183)*	0.00495 (0.00123)*
	N	0.00853 (0.00246)*	0.00416 (0.00132)*	0.002137(0.000931)*
1986 vs 1992:	Y	-0.00220 (0.00329)	-0.00136 (0.00226)	-0.0000378 (0.00149)
	C	-0.00380 (0.00310)	-0.00179 (0.00187)	-0.000730 (0.00126)
	N	-0.02699 (0.002689)*	-0.0149 (0.00147)*	-0.00787 (0.00102)*

[14]See the Appendix 1 for details.

[15]The 60% of 1986 income was chosen because 1986 was in the middle of the period of study, and because it was a peak year in terms of income after tax, consumption expenditures and necessities expenditures. This provides a high poverty line that includes most others considered.

1992 vs 1996:	Y			
	C	0.00969 (0.00313)*	0.00862 (0.00186)*	0.00482 (0.00126)*
	N	0.00865 (0.00285)*	0.00932 (0.00156)*	0.00592 (0.00109)*

The above results effectively illustrate the danger from relying exclusively on the income based poverty incidence. Between 1978 and 1982, the income based incidence significantly declined, despite there being a significant increase in poverty according the consumption and necessities based incidence. Furthermore, all higher order poverty indices indicate a significant decline in poverty. Between 1982 and 1986, the income based incidence significantly increased, while the consumption and necessities based incidence significantly declined. The disagreement in the trends of poverty carried through to the higher order indices, with income based poverty indices all rising while consumption and necessities based poverty declined. Between 1986 and 1992, no significant finding could be supported for the data except for a significant increase in poverty using the necessities indicator of household living standards. This observation seems quite unexpected as it is usual to think that necessities expenditures are likely to be less volatile than income or consumption expenditures. Finally, between 1992 and 1996, the consumption and necessities based poverty indices all indicate a *decline* in poverty - exactly opposite to the change indicated by the LICO incidence discussed in the introduction.

The problem with these statistics is that they only consider the poverty line at Y^p. This raises the question of whether the same outcome would be supported by all possible poverty lines below Y^p. The poverty dominance tests using the Kodde and Palm (1986) multiple comparison procedure allows this question to be addressed. The P values for the poverty dominance composite hypothesis are presented in table 4.2. As this is a symmetric test procedure, the tests for poverty dominance in both directions can be easily tested. Low P values reject the maintained hypothesis of dominance.

An examination of the results provides further insight into a number of anomalies that appeared in the simple tests for change in the FGT indices. For instance, the 1978 to 1982 comparison provided the anomalous result of a statistically significant increase in the income based poverty incidence, while higher level poverty indices calculated on income showed a significant decline in poverty.

Table 4.2 - P Values for the Poverty Dominance Tests

Indicator	Null Hypothesis	1st Order	2nd Order	3rd Order
Income	1982 ≻ 1978	1.25 e -04	3.25 e -14	1.18 e -14
	1978 ≻ 1982	4.39 e -08	0.989	1.00
Consumption	1982 ≻ 1978	1.37 e -06	1.78 e -06	8.40 e -07
	1978 ≻ 1982	0.745	0.986	1.00
Necessities	1982 ≻ 1978	2.01 e -13	2.93 e -13	1.15 e -13
	1978 ≻ 1982	0.711	0.972	1.00
Income	1986 ≻ 1982	2.31 e -3	2.95 e -3	1.61 e -3
	1982 ≻ 1986	0.752	0.990	1.00
Consumption	1986 ≻ 1982	0.745	0.985	1.00
	1982 ≻ 1986	9.48 e -7	1.20 e -6	5.59 e -7
Necessities	1986 ≻ 1982	0.707	0.968	1.00
	1982 ≻ 1986	1.68 e -6	2.44 e -6	1.53 e -6
Income	1992 ≻ 1986	4.12 e -4	5.67 e -23	1.00
	1986 ≻ 1992	0.5347	1.026 e -19	3.25 e -5
Consumption	1992 ≻ 1986	9.12 e -3	5.19 e -37	1.00
	1986 ≻ 1992	0.3045	8.09 e -31	6.86 e - 4
Necessities	1992 ≻ 1986	3.19 e -27	4.19 e -27	1.62 e -27
	1986 ≻ 1992	0.705	0.977	1.00
Income	1996 ≻ 1992			
	1992 ≻ 1996			
Consumption	1996 ≻ 1992	0.736	0.984	1.00
	1992 ≻ 1996	7.15 e -07	9.80 e -07	4.78 e -07
Necessities	1996 ≻ 1992	0.720	0.985	1.00
	1992 ≻ 1996	6.21 e -12	8.84 e -12	3.86 e -12

The first order poverty dominance test on income shows that no dominance can be found, ruling out the unambiguous decline in poverty suggested by the simpler tests. In other words, the direction of change in poverty incidence depends on where the poverty line is drawn. By contrast, the income data does not reject second and third order dominance of 1978 over 1982, suggesting that poverty rose when measured with higher order poverty indices. The data on consumption and necessities expenditures is completely consistent with the earlier finding of an increase in poverty over the 1978 to 1982 period. The uncontroversial conclusion that comes out of this evidence is that poverty increased over this first period.

For the comparisons of 1982 to 1986, the dominance results reinforce the earlier statistical findings. The clear dominance result using income indicates that any poverty line at or below Y^p will show that any FGT poverty index will register an increase in poverty. Consistent with the earlier tests, consumption and necessities data shows the opposite - an unambiguous decline in poverty. The irreducible contradiction leaves the analyst with the problem of settling on a preferred indicator if concrete conclusions are to be made about the direction of change in poverty over the 1982-86 period.

The 1986 to 1992 comparisons show a peculiar result where for income and consumption, first and third order dominance seem to indicate two different directions of poverty change. Most analysts would probably opt for the higher order dominance result as being the more influential as it has a correspondence to the axiomatically preferable FGT3 poverty index. For income and consumption the evidence rejects 3rd order dominance of 1986 over 1992, but fails to reject 3rd order dominance of 1992 over 1986, leading to the conclusion that poverty decreased. For necessities, the evidence points in the opposite direction to an increase in poverty. Again such a contradiction can only be resolved by deciding on a preferred indicator.

Finally, the 1992 to 1996 evidence on consumption and necessities expenditures both indicate a decline in poverty, contradicting the increase in the LICO incidence. One conclusion is that the new pro-cyclic rise in poverty would appear to be simply a strange artefact of the LICO methodology. Finally, the secular increase in the LICO poverty incidence apparent in figure 1.1 finds no consonance with the dominance rankings.

5. Conclusions

The use of the LICO poverty incidence in tracking poverty in Canada suffers from too many flaws to be considered reliable. Statistics Canada's LIM methodology avoids the problems of the LICO methodology by being simpler. It is easier to correctly interpret, and it does not rely on a shaky econometric foundation. The greatest advantage of the LIM methodology might well be the ease with which it can be extended to examine the implications from changing: assumptions made in constructing the adult equivalence scale, the cut-off percentage, and the measure of household resources. The RLIM, RLCM, and RLNM measures examined here, indicate that changing these assumptions can have a profound impact. Not just the measured level, but even the direction of change can depend on small, but reasonable changes in assumptions.

Adding to the confusion of determining the direction of change in poverty is the consideration of higher order, distribution sensitive poverty indices. With all three indicators considered here, there were instances where the incidence measure went in one direction while the other, more axiomatically sound indices went in another. What seems clear from this is that poverty incidence provides an unreliable indication of changes to poverty.

The poverty dominance approach can reduce some of this ambiguity by providing criteria under which poverty can be said to have changed for any reasonable poverty line. This obviates the need for checking for the robustness of different poverty line assumptions. The results indicate that poverty rankings are often possible and can serve the useful role of making robust poverty rankings. The only proviso is that a debate must take place on the appropriateness of various indicators of the resources available to households. Unless a consensus emerges on this, no strategy can solve the measurement problem.

To see why this is a serious problem, consider the question of whether the period between 1978 and 1996 has seen improvement in poverty. Using consumption expenditures as a guide, the tests for change in the FGT indices all indicate a reduction in poverty (see Table 3). Furthermore, these results are robust to changing the assumed adult equivalence scale. If, however, necessities expenditure is used as the measure of household resources, poverty shows unequivocal evidence of rise. Again this is true for all the FGT poverty indices and is robust to changes in the adult equivalence scale. Tests for poverty dominance confirms these opposite findings, leaving a deep

confusion about whether any progress has been made in the fight against poverty.

Table 5.1 Comparing Poverty in 1978 and 1996

		$FGT0_A$ - $FGT0_B$	$FGT1_A$ - $FGT1_B$	$FGT2_A$ - $FGT2_B$
ε=0.4	Y			
	C	-0.00442 (0.00305)	0.00470 (0.00188)*	0.00504 (0.00126)*
	N	-0.0339 (0.00264)*	-0.0107 (0.00143)*	-0.00410 (0.00100)
ε=0.5	Y			
	C	0.00445 (0.00303)	0.00997 (0.00183)*	0.00673 (0.00123)*
	N	-0.0238 (0.00259)*	-0.00768 (0.00138)*	-0.00297 (0.000975)*
ε=0.6	Y			
	C	0.0175 (0.00302)*	0.0168 (0.00180)*	0.0110 (0.00122)*
	N	-0.0143 (0.00263)*	-0.00441 (0.00140)*	-0.00146 (0.000988)*

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Appendix 1.

Given a common partitioning of the range space of two distributions into k mutually exclusive and exhaustive intervals of length d_j, j=1,..,k with respective relative frequency vectors p_A and p_B, define two matrices as follows:

$$I_f = \begin{Vmatrix} 1 & 0 & 0 & . & . & 0 \\ 1 & 1 & 0 & . & . & 0 \\ 1 & 1 & 1 & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 1 & 1 & 1 & . & . & 1 \end{Vmatrix}$$

$$I_F = .5 \begin{Vmatrix} d_1 & 0 & 0 & . & . & 0 \\ d_1+d_2 & d_2 & 0 & . & . & 0 \\ d_1+d_2 & d_2+d_3 & d_3 & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ d_1+d_2 & d_2+d_3 & d_3+d_4 & . & . & d_k \end{Vmatrix}$$

discrete empirical analogues can be developed to contemplate i'th order dominance by orienting test statistics around a null of no dominance viz:

$$H_0\colon I_F^{i-1} I_f (p^A - p^B) = 0 \ \textit{against}\ H_1\colon I_F^{i-1} I_f (p^A - p^B) \leq 0\colon$$
$$(\textit{note}\ H_2\colon I_F^{i-1} I_f (p^A - p^B) \leq \wedge \geq 0 \rightarrow \textit{indeterminacy})$$

where, in each case, under the alternative strict inequality must hold for at least one element of the vector. Let v be the difference between two k dimensioned empirical relative frequency vectors of households from two independent samples falling into the k mutually exclusive and exhaustive categories then, under the null hypothesis that the two samples come from a common distribution, it is known that v is asymptotically distributed $N(0,\Omega)$ where:

$$\Omega = \frac{n^A + n^B}{n^A n^B} \begin{Vmatrix} p_1(1-p_1) & -p_1p_2 & . & . & -p_1p_k \\ -p_2p_1 & p_2(1-p_2) & . & . & -p_2p_k \\ . & . & . & . & . \\ . & . & . & . & . \\ -p_kp_1 & -p_kp_2 & . & . & p_k(1-p_k) \end{Vmatrix}$$

Here the p_i, i = 1,..,k correspond to the probabilities under the null of a randomly selected household falling into the i'th category and the n^A and n^B are the sample sizes from the respective distributions. Stochastic dominance hypotheses can be examined in the context of $v^i = I_F^{i-1} I_f v$ for i=1,..,3 which, for suitably specified partitions, also have well defined asymptotically normal distributions (see Anderson (1996)). In practice, the pooled sample is employed to determine the fractiles (employed in calculating I_F) in the case of predetermined p, or the vector p (employed in calculating Ω) in the case of a predetermined partition of Y. The tests involve multiple comparison procedures which have been worked out (Richmond (1982)) and employed in the context of Lorenz curve ordinate confidence regions (Beach and Richmond (1985)) and require the use of the Studentized Maximum Modulus Distribution (Stoline and Ury (1979))[16]. Following the convenient convention of Bishop et. al. (1989) the hypothesis of dominance of distribution A over distribution B requires that no element of the appropriate vector v is significantly greater than 0 whilst at least one element is significantly less. Since the test is perfectly symmetric, dominance of B over A requires that no element of v is significantly less that 0 whilst at least one is significantly greater. In the event that v has both significantly negative and positive elements the comparison falls into the inconclusive region and no unambiguous statements regarding relative welfare can be made. In the event that no elements of v are significant the null of no stochastic dominance is not rejected.

There is a considerable literature on the joint testing of inequality restrictions (see Kodde and Palm (1986) and Wolak (1989) and references therein). Letting v_{iw} be the inequality constrained estimate of the vector v_i and let $(\Omega_{vi})^+$ be the Moore- Penrose inverse of the covariance matrix of v_i, then for:

$$W = (v_i - v_{iw})'(\Omega_{v_i})^+(v_i - v_{iw})$$

the distribution of W is such that:

[16] Appropriately normed quadratic forms of v^1, v^2 and v^3 would yield identical χ^2 statistics however this is not the case for multiple comparison procedures involving these vectors. Note that the nature of the matrix that transforms v into v^1 yields a value of the last element of v^1 that is identically 0.

$$P(W \geq c) = \sum_{i=0}^{k-1} P(\chi^2_i \geq c) w(k, k-i, \Omega_{v_i})$$

where w(k,k-i,Ω) is a weight function corresponding to the probability that v, with covariance matrix Ω, has k-i-1 of its k-1 independent elements positive. This weight function is complex to compute and closed form expressions only exist for k-1 up to 4, however following the suggestion in Wolak (1989) they can readily be approximated via pseudo normal random number generation.

Appendix 2

This may be seen by observing that the difference in the headcount measure for two distributions and a given poverty line is simply $F^1_A(Y^p) - F^1_B(Y^p)$. Note the depth of poverty measure (often referred to as the normalised deficit) for distribution j, D_j, is normally expressed as:

$$D_j = \frac{1}{Y^p} \int_0^{Y^p} (Y^p - y) dF_j(y)$$

A "well known result" in mathematics (see O'Brien (1984)) states that:

$$\int_0^{Y^p} (Y^p - y) dF_j(y) = F^2_j(Y^p)$$

thus it follows that $D_A - D_B$ may be written as $(Y^p)^{-1} (F^2_A(Y^p) - F^2_B(Y^p))$ which is the last element of the second order stochastic dominance vector divided by the poverty level. Obviously tests of the change in the normalised poverty depth measure and the significance of the last element of this vector are equivalent. Further, this presents a ready interpretation of the latter as the change in average income of people in poverty.