CSLS Conference on the State of Living Standards and the Quality of Life in Canada

October 30 - 31, 1998   Château Laurier Hotel, Ottawa, Ontario

Centre for the
Study of Living Standards
Centre d'étude des
niveaux de vie

Taking Prices Seriously in the Measurement of Inequality

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Session 6B: Trends in Inequality in Canada
October 31   8:30 AM - 11:00 AM

1. Introduction

The measurement of income[1] inequality usually focusses on income, and understandably so. However, both prices and demographics affect individual well-being, and these are incorporated into most analyses of economic inequality in very simple and misleading ways. Adjustments for price changes over time and differences across area of residence are usually done by dividing income by an income-independent price index. Adjustments for demographic differences across households are usually done by dividing income by a price-independent equivalence scale[2]. In this paper, I will briefly describe when these procedures are appropriate, argue that they are not in general appropriate, and show what difference it makes in the assessment of Canada's changing distribution of income over 1978-1996 to allow for more complex adjustment procedures.

In particular, I look at two refinements to the measurement of income inequality: (1) I relax the restriction that price effects must come in through dividing income by an income-independent price index. This amounts to allowing rich families and poor families to respond differently to price changes over time; (2) I relax the restriction that demographic effects must come in through dividing income by a price-independent equivalence scale. This amounts to allowing equivalence scales to depend on relative prices. A price-dependent equivalence scale for a large family might increase with a food price increase and decrease with a rent price increase because food is not as shareable as rent. These refinements to our assessment of consumption inequality in Canada over 1978-1996 make a big difference. Standard methods lead the researcher to conclude that family consumption inequality increased by about 15% over 1978-1986 and was fairly constant thereafter. The more robust methods outlined above lead the researcher to conclude that family consumption inequality increased by only about 8% over 1978-1986 and was fairly constant thereafter.

[1] I will use the terms income, consumption and expenditure interchangeably throughout this paper, in deference to the large literature on income inequality. In the empirical work, I evaluate changes in the distribution of consumption on a bundle of goods.

[2] In empirical work on economic inequality, equivalence scales for demographic adjustment are almost always assumed to be income-independent as well as price-independent. In this paper, I consider only the relaxation of price-independence. See Donaldson and Pendakur (1998) for a discussion of income- and price-dependent equivalence scales.

2. Theory

Define the expenditure function $E(p, u, z)$ as the minimum amount of expenditure necessary to give a household with demographic characteristics z facing prices p a utility level of u. Define the indirect utility function $V(p, x, z)$ as the level of utility attained by a household with income x and demographic characteristics z facing prices p. Define z^R as reference vector of household characteristics, to be used as the reference for demographic adjustments. For this paper, z^R is the vector of characteristics associated with a 40 year old single childless adult. Define $V^R(p, x) = V(p, x, z^R)$ as the indirect utility function of the reference household type and $E^R(p, u) = E(p, u, z^R)$ as the expenditure function of the reference household type. Define $\widetilde{p}$ as a base vector of prices, to be used as a basis for price adjustments. For this paper, $\widetilde{p}$ is the price vector faced by residents of Ontario in 1986.

The measurement of inequality requires an income measure that has been adjusted for differences in the price regimes faced by different households and the demographic characteristics of different households. Saying that it is possible to "adjust" for differences across household types in the measurement of inequality requires that the adjusted distribution with no differences in characteristics or price regimes is normatively equivalent to the unadjusted distribution with differences in characteristics and/or price regimes. When ethics are consequentialist, as they are under welfarist social evaluation, normative equivalence between two distributions is satisfied when the set of individual utilities in each distribution is identical. Thus, we define *adjusted income* denoted $y = Y(p, x, z)$ as the income that would give a household with reference characteristics facing base prices the same utility as a household with income x and characteristics z facing prices p. Adjusted income is defined implicitly by:

$$V^R(\widetilde{p}, y) = V^R(\widetilde{p}, Y(p, x, z)) = V(p, x, z). \tag{2.1}$$

A population of reference households facing base prices with incomes $\{Y(p, x, z)\}$ is normatively equivalent to a population of households with characteristics $\{z\}$ facing price vectors $\{p\}$ with incomes $\{x\}$. Given a population of individuals living in households with various demographics compositions and facing various price regimes, the researcher calculates an inequality index based on the adjusted income, $Y(p, x, z)$, assigned to each individual.

Without loss of generality, we can break the derivation of adjusted income into two steps: (1) adjustment for differences in demographic characteristics across households evaluated at current prices; and (2) adjustment for differences in price regimes across different households. I consider these two steps in turn.

2.1. Adjusting for differences in household characteristics

Define $x^e = X(p, x, z)$ as equivalent income which gives the income necessary for a reference household (with characteristics z^R) facing prices p to achieve the same utility level as a household with characteristics z facing prices p. The equivalent income function $X(p, x, z)$ is implicitly given by

$$V^R(p, x^e) = V^R(p, X(p, x, z)) = V(p, x, z) \tag{2.2}$$

and given by

$$X(p, x, z) = E^R(p, V(p, x, z)).$$

A population of reference households facing prices p with incomes $\{X(p, x, z)\}$ is normatively equivalent to a population of households with characteristics $\{z\}$ facing prices p with incomes $\{x\}$. If the researcher knows the equivalent income function $X(p, x, z)$, then to measure income inequality within a group of households facing a common price vector p, s/he would assign equivalent income x^e to each member of each household and measure income inequality on these equivalent income data.

The most common strategy in the measurement of inequality is to use an equivalent income function of the form

$$X(p, x, z) = \frac{x}{S(z)}. \tag{2.3}$$

Here, equivalent income is calculated by dividing income by a price-independent equivalence scale function $S(z)$.

Unfortunately, Blackorby and Donaldson (1993) show that if the equivalent income function is given by (2.3), then the expenditure function is very restricted and must be given by

$$E(p, u, z) = E^R(p, u)S(z). \tag{2.4}$$

Pendakur (1999) further shows that if the equivalent income function is given by (2.3) then expenditure share equations $w_j(p, x, z)$ are given by

$$w_j(p, x, z) = w_j^R(p, \frac{x}{S(z)}) \tag{2.5}$$

where $w_j^R(p,x) = w_j(p,x,z^R)$ give the expenditure share equations of the reference household. Expressing (2.5) in terms of $\ln x$, we get

$$w_j(p, \ln x, z) = w_j^R(p, \ln x - \ln S(z)). \tag{2.6}$$

Here, we see that household characteristics are highly restricted in the way they enter expenditure share equations. If (2.3) holds, then expenditure share equations in $\ln x$ must be identical across household types except for a horizontal translation of $\ln S(z)$.

There is a wealth of parametric (Blundell and Lewbel 1991; Dickens, Fry and Pashardes 1993; Pashardes 1995) and semiparametric (Gozalo 1997; Pendakur 1999) research that tests the restrictions given by (2.5), and this research rejects the hypothesis that these restrictions hold. This means that the price-independence of $S(z)$ cannot hold. That is, the equivalent income function must have a more complicated relationship with income than that given by (2.3).

This suggests that empirical work measuring income inequality with equivalence scaling of the form (2.3) may be misleading. In this paper, I relax the restriction that the equivalent income function is independent of prices by using an equivalent income function of the form

$$X(p,x,z) = \frac{x}{R(p,z)}. \tag{2.7}$$

Blackorby and Donaldson (1993) show that under certain conditions equivalent income functions of the form (2.7) are identifiable from demand data. In this paper, I estimate $R(p,z)$ from demand behaviour (see the Estimation Appendix for details), and use this estimated price-dependent equivalence scale to evaluate how the change in measured inequality responds to using an equivalent income function more general than the naive equivalent income function (2.3).

2.2. Adjusting for differences in price regimes

Consider a population of households with identical reference characteristics z^R facing prices $\{p\}$ with incomes $\{x\}$. Define $\widetilde{x} = D(p,x)$ as *deflated income* which gives the income necessary for a reference household facing a base prices $\widetilde{p}$ to achieve the same utility level as a reference household facing prices p. Deflated income is implicitly defined by:

$$V^R(p^R, \widetilde{x}) = V^R(p^R, D(p,x)) = V^R(p,x) \tag{2.8}$$

and given by

$$D(p,x) = E^R(\tilde{p}, V^R(p,x)). \tag{2.9}$$

A population of reference households facing base prices $\tilde{p}$ with a set of associated deflated incomes $\{D(p,x)\}$ is normatively equivalent to a population of reference households facing prices $\{p\}$ with incomes $\{x\}$. If the researcher knows the deflated income function $D(p,x)$, then to measure income inequality within a group of households with common reference characteristics facing different price vectors, s/he would assign deflated income $\tilde{x}$ to each household and measure income inequality on these deflated income data.

Note that a population of reference households facing prices $\{p\}$ with incomes $\{X(p,x,z)\}$ is normatively equivalent to a population of households with differing characteristics $\{z\}$ facing prices $\{p\}$ with incomes $\{x\}$. To simulateously allow for differences in household characteristics, the researcher could use a two-step process. First assign each member of each household equivalent income x^e. Second, compute adjusted income $y = Y(p,x,z)$ as

$$Y(p,x,z) = D(p,x^e) = D(p, X(p,x,z)) \tag{2.10}$$

or, substituting in for D and X,

$$Y(p,x,z) = E^R\left(\tilde{p}, V^R\left(p, E^R\left(p, V(p,x,z)\right)\right)\right). \tag{2.11}$$

The most common form chosen for $D(p,x)$ in the measurement of income inequality is

$$D(p,x) = \frac{x}{I(p)} \tag{2.12}$$

where $I(p)$ is an income-independent price index. Here, deflated income is calculated by dividing income by the price index $I(p)$. In this case, if the researcher is using a relative inequality index (which is insensitive to scalings of the income data), it is not even necessary to use the deflated income measure: relative inequality will be the same whether calculated on a set of income data or a set of deflated income data.

Unfortunately, Diewert (REF) shows that if the deflated income function is given by (2.12) the (reference) expenditure function must be given by

$$E^R(p,u) = F(p)G(u). \tag{2.13}$$

and $I(p)$ is given by

$$I(p) = \frac{F(p)}{F(\tilde{p})}$$

This condition is called homotheticity and is a very tight restriction on preferences. If expenditure functions are given by (2.13) then expenditure share equations must be given by

$$w_j^R(p,x) = \frac{d \ln F(p)}{d \ln p_j}. \tag{2.14}$$

Here expenditure shares are independent of income.

A large body of empirical work suggests that expenditure shares for many commodities are highly dependent on income (see, for example, Banks, Blundell and Lewbel 1997). For example, poor households spend a larger share of their income on food than rich households. This suggests that the deflated income function might be related to income in a more complicated way than that given by (2.12).

If the researcher uses both the naive equivalent income function given by (2.3) and the naive deflated income function given by (2.12), then the expenditure function must be given by

$$E(p,u,z) = F(p)G(u)S(z). \tag{2.15}$$

This expenditure function is characterised by identically homothetic preferences for all household types. Here, expenditure share equations are given by

$$w_j(p,x,z) = \frac{d \ln F(p)}{d \ln p_j} \tag{2.16}$$

so that share equations are independent of income and household characteristics.

As noted above, in this paper, I relax the restriction that the equivalent income function take the form (2.3). I also relax the restriction that the deflated income function be dual to homothetic preferences and given by (2.12). I calculate $D(p,x)$ by estimating a nonhomothetic Quadratic Almost Ideal (QUAI) demand system due to Banks, Blundell and Lewbel (1997) (see Estimation Appendix for details). Whereas homothetic demand systems require that expenditure shares for different goods are constant across x, the QUAI demand system allows expenditure shares to be quadratic in the log of x.

3. The Data

This analysis uses the 1978, 1982, 1984, 1986, 1990, 1992 and 1996 Family Expenditure Surveys (FES). These surveys were conducted by Statistics Canada and collect information on demographics, income and expenditures from five to ten thousand Canadian households in each survey year (Statistics Canada, various years). The universe for these surveys varied from year to year: the 1978, 1982, 1986, and 1992 surveys sampled from all urban and rural households in the ten provinces of Canada, while the 1984 and 1990 surveys sampled only from urban households in Canada's 15 largest Census Metropolitan Areas (CMAs). To maximize the number of survey years available and to avoid imputing rural consumption, I analyse data on urban residents only, unless otherwise specified.

The 1978, 1982, 1984 and 1986 surveys treat the "spending units" as the unit of analysis and the 1990, 1992 and 1996 surveys treat the household as the unit of analysis. In order to create a consistent sample across these units of analysis, I use only spending units and households comprised of a single economic family. An economic family is defined as an unattached individual, or a group of people, related by blood, marriage or adoption, who live together in a household. The FES data are weighted at the level of the family, so each individual in a family is assigned the family weight.

I use only economic families that fall into six household types: (1) single childless adults; (2) childless couples; (3) single parents with one child; (4) single with two children; (5) dual parents with one child; (6) dual parents with two children. A single childless adult aged 40 years old is the reference household type, denoted above as z^r.

I assess inequality in consumption of a bundle of five commodities: (1) food; (2) shelter, including rented and owned accomodation; (3) clothing for adults; (4) clothing for children; and (5) transportation. I use the Gini coefficient to measure inequality, and supply standard errors for estimated Gini coefficiencts following Barrett and Pendakur (1995).

Because the consumption flow from durables is smoother than the purchase path, I calculate imputed rental flows[3] for households with owned accomodation and imputed transportation flows[4] for households with automobile expenditures.

[3]I calculate imputed rental flows by taking predicted values from city- and year-specific regressions of rents on the number of bedrooms and bathrooms and income and income squared in housing units. The coefficients for these predictions are estimated from regressions using only rental units. The predicted values are applied to owned units. Thus, to the extent that owned accomodation is a better class of accomodation than rented accomodation, these imputed rental flows may underestimate the actual rental flow from owned accomodation.

[4]I calculate imputed transportation flows by taking predicted probabilities from city- and year-specific logit regressions of automobile purchase dummies on family income and family income squared. These probabilities are multiplied by predicted values from city- and year-

Imputed rental flows are added to and mortgage costs subtracted from shelter costs for households with owned accomodation. Imputed transportation flows are added and automobile purchase costs subtracted from transportation costs for households with automobile expenditures.

In order to assess the sensitivity of measured inequality to the relaxation of the restrictions given by (2.3) and (2.12), I use the following naive equivalent income and deflated income functions. The naive equivalent income function is given by

$$X(p, x, z) = \frac{x}{n^{1/2}} \tag{3.1}$$

where n is a an element of z equal to the number of household members. This equivalence scale lies roughly in the middle of the range of scales surveyed by Buhmann et al (1987), and is the same as that used by Atkinson (1994) in his study of income inequality in OECD countries. Each individual in a family is assigned equivalent consumption equal to the total family income or consumption divided by the square root of the number of household members.

As noted above the flexible equivalent income function is given by

$$X(p, x, z) = \frac{x}{R(p, z)}. \tag{3.2}$$

The functions $R(p, z)$ is Cobb-Douglas in prices and fully flexible with respect to the six household types noted above and also sensitive to the age of the household head (see Estimation Appendix for details). Table 1 shows the estimated relative equivalence scale $R(p, z)$ for each of the six household types, evaluated for a 40 year old household head and at the price vector for Ontario 1986.

Table 1: Estimated Price-Dependent Equivalence Scales

	Couples	Single Parents		Dual Parents	
	0 Kids	1 Kid	2 Kids	1 Kid	2 Kids
$R(p, z)$	1.276	1.500	2.296	1.927	2.097
Std Error	*0.089*	*0.197*	*0.423*	*0.213*	*0.256*

The naive deflated income function is given by (2.12). Because relative inequality indices are homogeneous of degree 0, relative inequality indices do not change if a common income-independent price index is applied to all observations in a given year. Thus, from the point of view of inequality measurement using

specific regressions of automobile purchase prices on family income and family income squared to get imputed transportation flows.

relative inequality indices or Lorenz dominance criteria, when the researcher requires the deflated income function to take the form (2.12), it is not necessary to deflate the income measure.

The flexible deflated income function is that associated with Banks, Blundell and Lewbel's (1997) Quadratic Almost Ideal (QUAI) model. This model is characterised by expenditure share equations that are quadratic in the log of total expenditure (see Estimation Appendix for details). The (reference) indirect utility function associated with the QUAI model is

$$V^R(p,x) = \left(\left(\frac{\ln x - \ln a(p)}{b(p)} \right)^{-1} - q(p) \right)^{-1} \tag{3.3}$$

where $a(p)$, $b(p)$ and $q(p)$ are homogeneous of degrees one, zero and zero, respectively. The key feature of the QUAI model is that it allows for expenditure share equations that are quadratic in the log of total expenditure and has been shown to fit the data tolerably well (for details, see Banks, Blundell and Lewbel 1997).

4. Results

Table 2 presents estimated Gini coefficients for four measures of individual well-being. They are all functions of family expenditure, x, defined above as the sum of food, shelter, clothing and transportation expenditures. I note that I use the terms "income" and "expenditure" interchangeably to refer to family expenditure. The four measures are:

- Naive equivalent income, x_N^e, defined as family expenditure divided by a naive equivalence scale equal to the square root of household size. This equivalent income is defined by equation (3.1).
- Flexible equivalent income, x_F^e, defined as family expenditure divided by an equivalence scale that depends on prices. This equivalent income is defined by equation (3.2).
- Deflated naive equivalent income, $D(x_N^e)$,which is equal to naive equivalent income x_N^e deflated to Ontario 1986 prices using equation (**??**).
- Deflated equivalent income (1), $D(x_F^e)$, which is equal to flexible equivalent income x_F^e deflated to Ontario 1986 prices using equation (**??**).

I further note that the most common approach to the measurement of inequality is to calculate an inequality index on observations of naive equivalent income.

Table 2: Gini Coefficients for Family Consumption, Canada, 1978-1996

Year	1978	1982	1984	1986	1990	1992	1996
x_N^e	0.1932	0.2040	0.2079	0.2217	0.2208	0.2236	0.2238
std err	*0.0028*	*0.0024*	*0.0033*	*0.0028*	*0.0037*	*0.0029*	*0.0026*
x_F^e	0.2140	0.2224	0.2238	0.2366	0.2345	0.2401	0.2366
std err	*0.0031*	*0.0026*	*0.0036*	*0.0030*	*0.0039*	*0.0031*	*0.0028*
$D(x_N^e)$	0.1901	0.1964	0.2027	0.2165	0.2117	0.2153	0.2143
std err	*0.0028*	*0.0023*	*0.0032*	*0.0028*	*0.0036*	*0.0028*	*0.0025*
$D(x_F^e)$	0.2108	0.2140	0.2181	0.2306	0.2245	0.2308	0.2256
std err	*0.0031*	*0.0025*	*0.0035*	*0.0029*	*0.0038*	*0.0030*	*0.0027*

Figure 1 shows the Gini coefficients for these measures over 1978-1996. Figure 2 shows how the Gini coefficient for these four measures changes over time in absolute terms, treating 1978 as the base year. Figure 3 shows the how the Gini coefficient for these four measures changes over time in proportionate terms, treating 1978 as the base year.

The first row of Table 1 and the dotted lines with empty markers in Figures 1, 2 and 3 give the Gini coefficients found when we use standard (naive) method of adjusting for demographics and prices. Looking at Figure 1, we see inequality is rising over 1978-1986, falling to 1990 and rising through 1996. The 1986 peak is about the same as the 1996 peak, and these peaks are insignificantly different from each other. Betweeen 1978 and 1996, the Gini coefficient computed off of the naive income measure rose just over three percentage points, most of which occurred during 1978-1986. This large increase in measured inequality is consistent with findings on the evolution of consumption inequality in Canada using the naive adjustment approach (see Pendakur 1998).

Comparing these estimates to those with naive demographic adjustment and flexible price adjustment (the dotted lines with filled markers in the Figures), we see that the Gini coefficient falls by a little less than one percentage point over the entire period. This is due to the fact that adjusting for price differences accounts for the fact that many wealthy families live in areas where the cost of living is high. However, more than just the measured level of inequality changes when we adjust for prices. Looking at Figure 2, we see clearly that in periods of income growth (1986-1990 and 1992-1996), the price adjustment pushes down estimated growth in the level of inequality. In particular, while 1992-1996 is a period of rising inequality on the naive measure, it is a period of declining inequality on the price-adjusted measure. Turning to Figure 3, we see that the proportionate growth in inequality is about twelve percent with flexible price adjustment, compared to about sixteen percent with naive price adjustment. Most of this differs is over the period 1992-1996, a period during which price increases hurt the rich relatively more than the poor.

Turning now to the case where we use flexible demographic and price adjustments (heavy lines with filled markers in the Figures), Figure 1 shows that the flexible demographic adjustment also makes a big difference at the margin. In comparison to using a naive demographic adjustment, using a price-dependent equivalence scale pushes up the measured level of inequality by between one and two percentage points. Notably, it pushes measured inequality up by two percentage points at the beginning of the period and only one percentage point at the end of the period. Looking at Figure 2, we see that whereas inequality measured with flexible price adjustment and naive demographic adjustment rose by 2.4 percentage points, inequality measured with flexible price and demographic adjustment rose by only 1.6 percentage points. Figure 3 gives the proportionate changes in the Gini Coefficient. Whereas the Gini coefficient with naive demographic adjustment (and flexible price adjustment) rose by about twelve percent, the Gini coefficient with flexible demographic and price adjustment rose by only eight percent.

We can interpret the effect of flexible demographic adjustment on measured inequality in terms of the how equivalence scales vary across regions due to price differences across regions. In particular, equivalence scales for families with children tend to be smaller in Ontario and BC than in other regions. This is because the price of shelter, a highly shareable commodity, is higher in BC and Ontario than in other regions. Since residents of Ontario and BC are on average richer than residents of other regions, the flexible demographic adjustment pushes measured inequality downwards. Further, relative price changes over the period, especially the increases in the relative price of shelter in BC and Ontario in the early 1980s and early 1990s, sharpened this effect over time.

Turning to Figure 3, we see that the combined effect of flexible price and demographic adjustment substantially reduces our assessment of the growth in consumption inequality over the early 1980s. Whereas the naive method shows a sixteen percent increase in the level of inequality over 1978-1986, the estimates using flexible price and demographic adjustment show only an eight percent increase in the level of inequality over this period. Further, whereas the naive method shows a statistically significant increase in inequality over 1992-1996, the estimates using flexible price and demographic adjustment show a statistically significant decrease in inequality over 1992-1996.

Figure 4 presents estimated Gini coefficients using naive price and demographic adjustments for five regions of Canada, and Figure 5 presents estimated Gini coefficients using flexible price and demographic adjustments for five regions of Canada. Looking at Figure 4, two conclusions emerge from the naive approach. First, the trend increase in inequality does not seem to be driven by any one region, or by inter-region inequality. It seems that all regions saw increasing inequality on the order of at least one-tenth over 1978-1996. Second, the cyclical path of inequality over the business cycle seems to be different across regions. In particular, Ontario and BC saw decreasing or flat inequalty over growth periods 1978-1982 and 1992-1996, but other regions saw increasing inequality over these periods.

Turning to Figure 5, which uses flexible price and demographic adjustments, we modify our conclusions in two ways. First, although most regions have seen an increase in inequality over 1978-1996, this growth is much smaller than that depicted in Figure 4. Second, BC is a clear outlier when flexible price and demographic adjustments are used. Whereas using flexible adjustment pushes down our estimate of the increase in inequality in Canada as a whole over 1978-1996, it pushes up our estimate of the increase in inequality in BC over 1978-1996. Using flexible adjustment, inequality rose in most regions by about one and one-half percentage points. However, in BC, inequality rose by over four percentage points, concentrated in the period 1990-1992.

5. Conclusions

The measurement of inequality is usually characterised by the use of an inequality index and an adjusted income measure. The adjusted income measure is usually calculated by adjusting for demographic differences with a price-independent equivalence scale and adjusting for price differences with an income-independent price index. Relaxation of these independence restrictions allows for more flexible price and demographic adjustment and leads the researcher to substantively different conclusions about the path of consumption inequality in Canada over 1978-1996. Standard methods lead the researcher to conclude that family consumption inequality increased by about 15% over 1978-1986 and was fairly constant thereafter. The more robust methods outlined above lead the researcher to conclude that family consumption inequality increased by only about 8% over 1978-1986 and was fairly constant thereafter.

6. Estimation Appendix

To estimate the price-dependent equivalence scale $R(p,z)$ and the flexible deflated income function, $D(p,x)$, I specify functional forms for $R(p,z)$ and $V^R(p,x)$ and estimate by maximum likelihood. I estimate $R(p,z)$ as a Cobb-Douglas in prices as follows:

$$\begin{aligned} R(p,z) &= R_0(z)\prod_{k=1}^{N} p_k^{R_k(z)} \\ \text{with } \sum_{k=1}^{N} R_k(z) &= 0. \end{aligned} \tag{6.1}$$

The functions $R_m(z)$ are linear indices of a vector of 5 household type dummies z^l and the age of the household head:

$$R_m(z) = \sum_{l=1}^{5} R_m^l z^l + R_m^{age}\ln(\text{age of household head}) + R_m^{age^2}\ln(\text{age of household head})^2.$$

The age of household head is divided by 40, the average head age in the sample so that $R_m(z) = \sum_{l=1}^{5} R_m^l z^l$ for households with heads aged 40 years old.

Because childless households do not demand children's clothing, I code children's clothing expenditures as zero for these households (4% of childless households have nonzero children's clothing expenditures, presumably due to gift-giving). Further, I restrict the demand system so that childless households have zero demands children's clothing.

The flexible deflated income function is that associated with Banks, Blundell and Lewbel's (1997) QUAI model. This model is characterised by expenditure share equations that are quadratic in the log of total expenditure. The (reference) indirect utility and expenditure functions associated with the QUAI model is

$$V^R(p,x) = \left(\left(\frac{\ln x - \ln a(p)}{b(p)}\right)^{-1} - q(p)\right)^{-1} \tag{6.2}$$

and

$$\ln E^R(p,u) = \ln a(p) + \frac{b(p)u}{1-q(p)u} \tag{6.3}$$

where $a(p)$, $b(p)$ and $q(p)$ are given by

$$\begin{aligned}
\ln a(p) &= a_0 + \sum_{j=1}^{m} a_j \ln p_j + \frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m} a_{jk} \ln p_j \ln p_k \\
\text{with } \sum_{j=1}^{m} a_j &= 1,\ \sum_{j=1}^{m} a_{jk} = 0 \forall k,\ a_{jk} = a_{kj}\ \forall j,k; \\
\ln b(p) &= \sum_{j=1}^{m} b_j \ln p_j + \frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m} b_{jk} \ln p_j \ln p_k \\
\text{with } \sum_{j=1}^{m} b_j &= 0,\ \sum_{j=1}^{m} b_{jk} = 0 \forall k,\ b_{jk} = b_{kj}\ \forall j,k; \\
q(p) &= \sum_{j=1}^{m} q_j \ln p_j \text{ with } \sum_{j=1}^{m} q_j = 0.
\end{aligned}$$

Given these choices for $a(p)$, $b(p)$ and $q(p)$, the adjusted income function $Y(p,x,z)$ is given by

$$\begin{aligned}
\ln Y(p,x,z) &= \ln E^R(\widetilde{p}, x^e) \\
&= \ln E^R(\widetilde{p}, \frac{x}{R(p,z)}) \\
&= \ln a(\widetilde{p}) + \ln x - \ln R(p,z) - \ln a(p).
\end{aligned}$$

Defining $\widetilde{\ln x} = \ln x - \ln a(p)$, application of Roy's Identity generates translated log-quadratic reference expenditure share equations $W_j^r(p, \ln x)$ as follows:

$$W_j^r(p,\ln x) = \frac{\partial \ln a(p)}{\partial \ln p_j} + \frac{\partial \ln b(p)}{\partial \ln p_j}\widetilde{\ln x} + \frac{\frac{\partial q(p)}{\partial \ln p_j}}{b(p)}\widetilde{\ln x}^2. \tag{6.4}$$

Substituting in the relevant derivatives and elasticities,

$$W_j^r(p,\ln x) = a_j + \sum_{k=1}^{N} a_{jk} \ln p_k + b_j \widetilde{\ln x} + \frac{q_j}{b(p)}\widetilde{\ln x}^2. \tag{6.5}$$

Pendakur (1999) shows that if a price-dependent equivalence scale gives an exact cost-of-demographics index, the equivalence scale $R(p, z)$ fully defines nonreference expenditure share equations in terms of reference share equations as follows:

$$W_j(p, \ln x, z) = a_j + \sum_{k=1}^{N} a_{jk} \ln p_k + b_j \left(\widetilde{\ln x} - \ln R(p, z)\right) + \frac{q_j}{b(p)} \left(\widetilde{\ln x} - \ln R(p, z)\right)^2 + R_j(z). \tag{6.6}$$

Blundell, Duncan and Pendakur (1998) find that this specification for share equations and demographic effects stands up quite well in comparison with a nonparametric alternative.

After adding an error term to each equation, this equation system was by maximum likelihood using TSP. Notable issues include:

- Cross-equation correlations in errors are estimated.
- The errors are assumed homoskedastic across x and z.
- I do not use a GMM estimator to allow for endogenous x. Many researchers allow for endogeneity in income because their expenditure data are available for very short periods (eg, British expenditure data is based on two-week expenditures) and therefore suffer from durable goods contamination. Canadian expenditure data cover one-year expenditures.
- These results are similar to Pashardes (1995) results based on American quarterly expenditure data.
- Detailed parameter estimates are available from the author on request.

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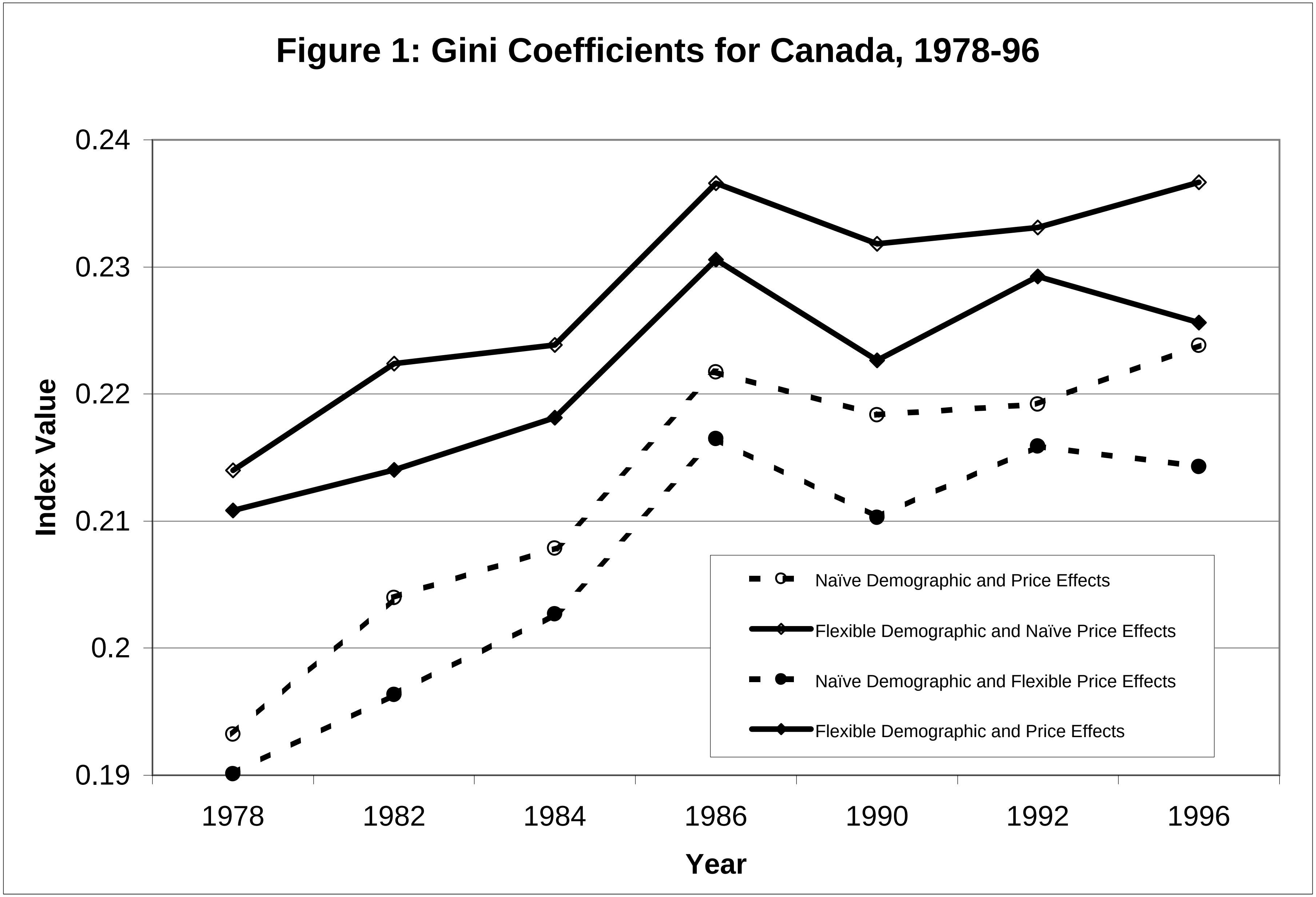
Figure 1: Gini Coefficients for Canada, 1978-96
Index Value
0.24
0.23
0.22
0.21
0.2
0.19
1978
1982
1984
1986
1990
1992
1996
Year
Naïve Demographic and Price Effects
Flexible Demographic and Naïve Price Effects
Naïve Demographic and Flexible Price Effects
Flexible Demographic and Price Effects

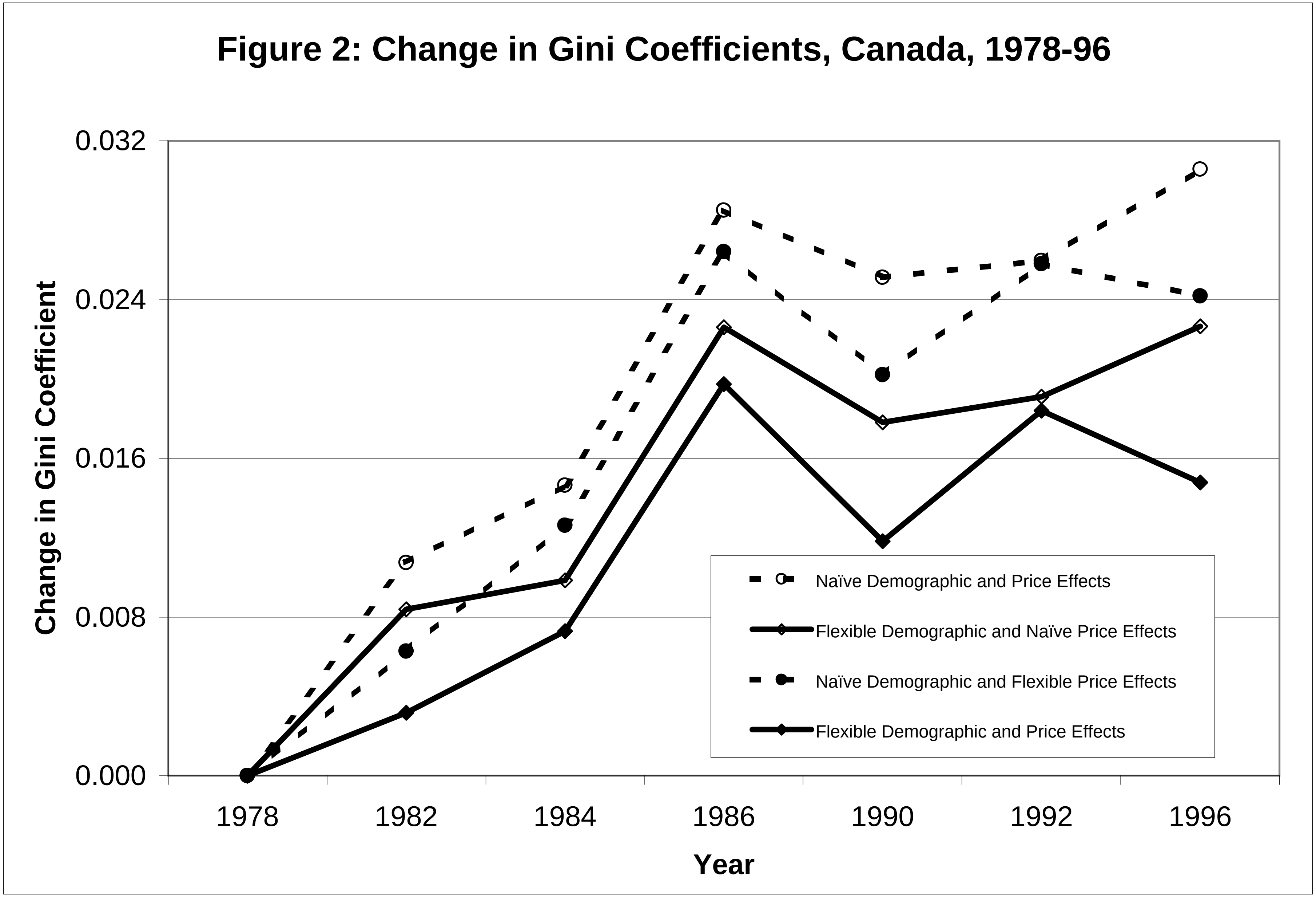
Figure 2: Change in Gini Coefficients, Canada, 1978-96
Change in Gini Coefficient
0.032
0.024
0.016
0.008
0.000
1978
1982
1984
1986
1990
1992
1996
Year
Naïve Demographic and Price Effects
Flexible Demographic and Naïve Price Effects
Naïve Demographic and Flexible Price Effects
Flexible Demographic and Price Effects

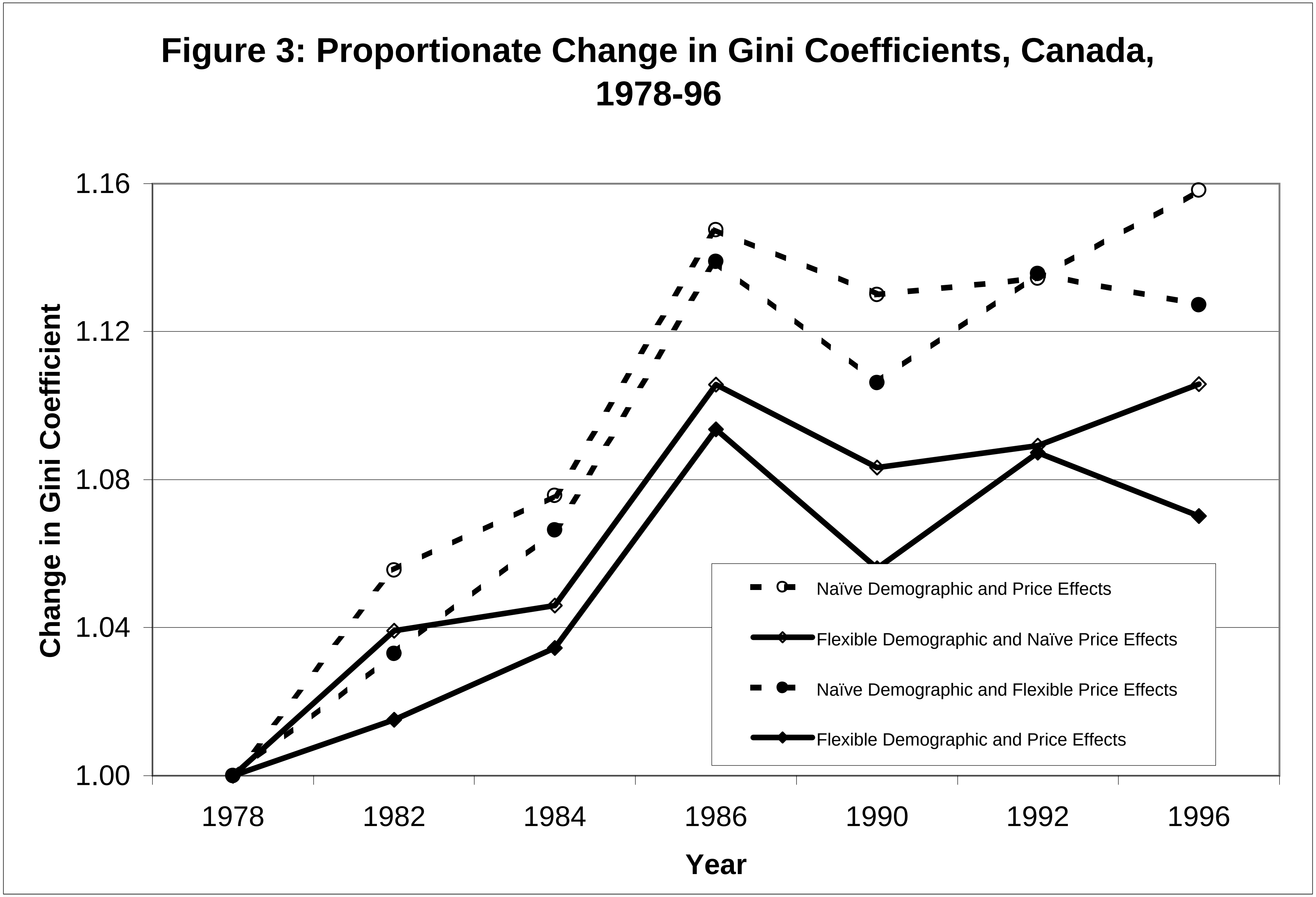
Figure 3: Proportionate Change in Gini Coefficients, Canada, 1978-96
Change in Gini Coefficient
1.16
1.12
1.08
1.04
1.00
1978
1982
1984
1986
1990
1992
1996
Year
Naïve Demographic and Price Effects
Flexible Demographic and Naïve Price Effects
Naïve Demographic and Flexible Price Effects
Flexible Demographic and Price Effects

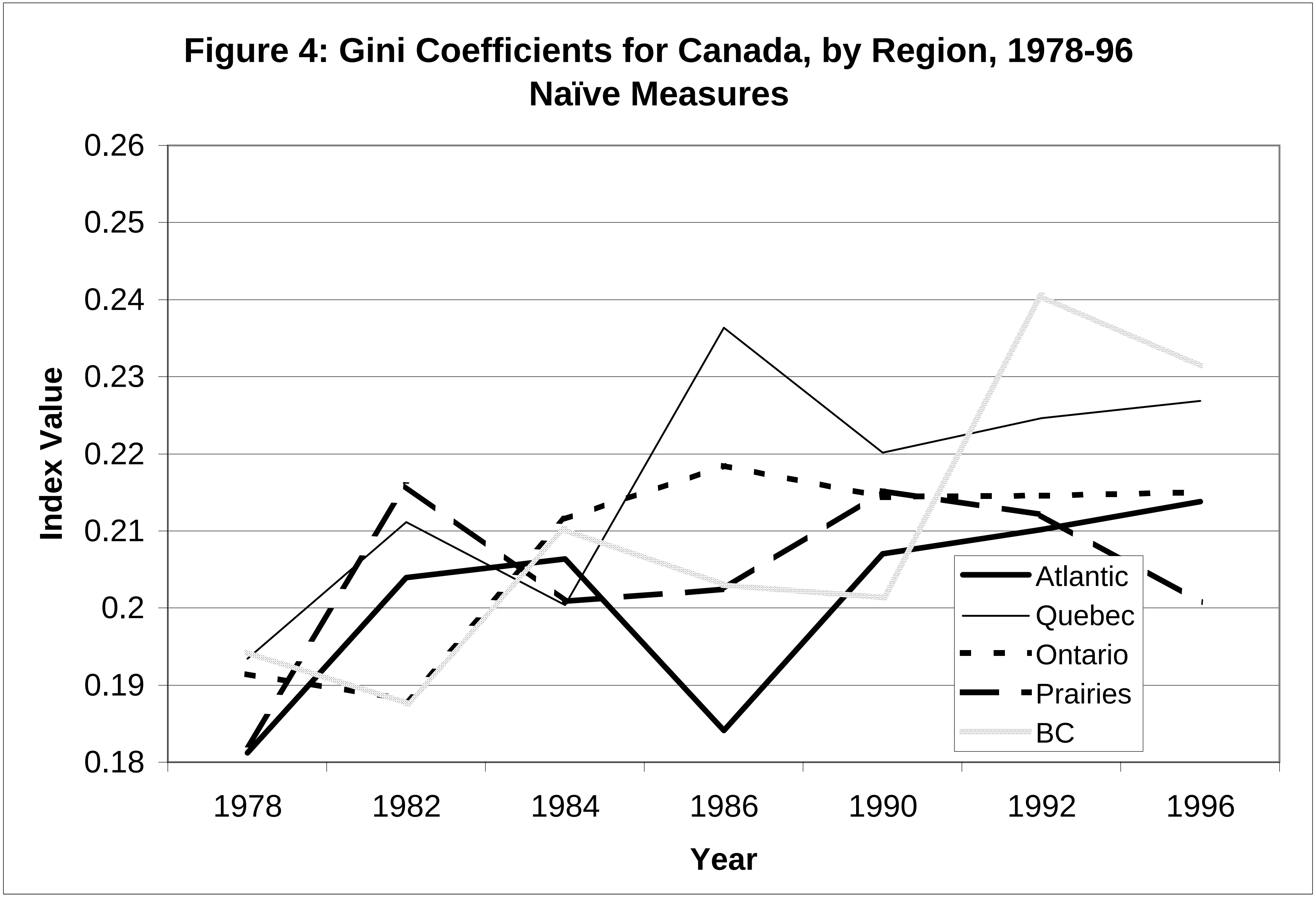
Figure 4: Gini Coefficients for Canada, by Region, 1978-96
Naïve Measures
Index Value
0.26
0.25
0.24
0.23
0.22
0.21
0.2
0.19
0.18
1978
1982
1984
1986
1990
1992
1996
Year
Atlantic
Quebec
Ontario
Prairies
BC

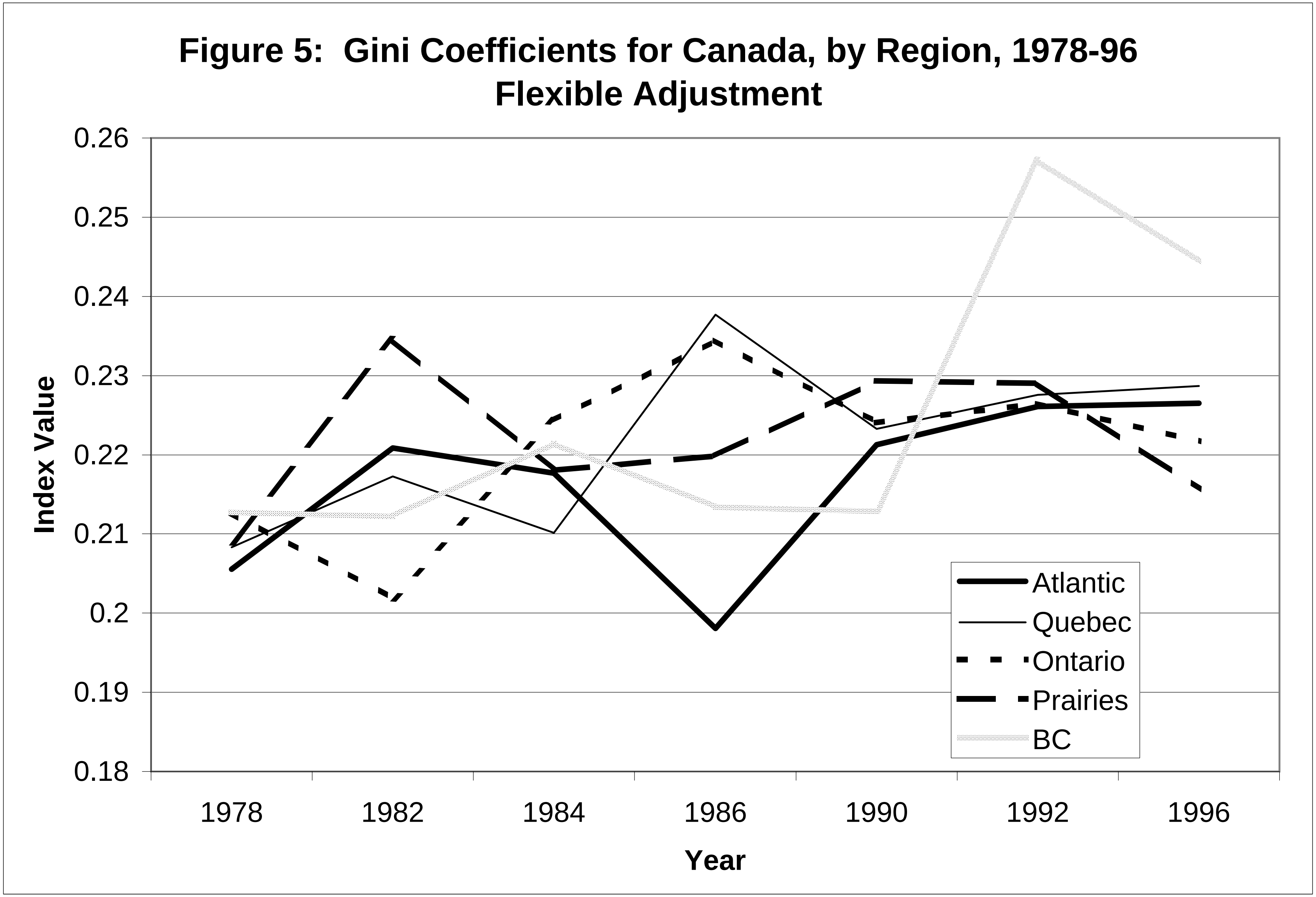

Figure 5: Gini Coefficients for Canada, by Region, 1978-96
Flexible Adjustment
Index Value
0.26
0.25
0.24
0.23
0.22
0.21
0.2
0.19
0.18
1978
1982
1984
1986
1990
1992
1996
Year
Atlantic
Quebec
Ontario
Prairies
BC