Real GDP, Real GDI, and Trading Gains: Canada, 1981-2005

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ABSTRACT

Real gross domestic product (GDP) fails to account for the trading gains and losses that result from changes in the terms of trade and in the real exchange rate (the price of tradables relative to the price of nontradables). Canada has enjoyed vast improvements in its terms of trade over recent years and there is a growing suspicion that real GDP has done an inadequate job at reflecting the resulting increases in real value added and real gross domestic income (GDI). Superlative measures of the terms of trade and real exchange rate effects confirm this view, with the trading gains adding up to 4.8 per cent of GDP between 2002 and 2005.

CANADA HAS EXPERIENCED SUBSTANTIAL improvements in its terms of trade over the past few years.[2] As established by international trade theory, an improvement in the terms of trade, other things equal, is income and welfare enhancing. There is a growing suspicion, however, in Canada and elsewhere, that real gross domestic product (GDP) does an inadequate job at capturing this increase in real value added. The following quote is quite representative:

> "[...] some economists take the view that official GDP data have significantly understated Canada's growth rate over the past three years by not taking account of the vast improvement, until the past month or two, in the terms of trade," Bernard Simon in the *Financial Times*, July 12, 2006.

As argued by Diewert and Morrison (1986) in a seminal article, an improvement in the terms of trade is similar to technological progress. The country essentially gets more for less. Yet, unlike a technological advance, a terms of trade improvement is treated by the national accounts as a price phenomenon, rather than as a real development. An increase in export prices or a drop in import prices, for instance, will not only tend to increase nominal GDP, but it will also raise the GDP deflator, leaving real GDP little changed, even though real value added and real income must unambiguously have increased.[3]

Canada is not an isolated case. Australia, on the wave of a massive increase in commodity prices in recent years, has been facing a similar

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2 Thus, the export price index increased by 3.4 per cent between 2002 and 2005, whereas the import price index fell by 9.8 per cent over the same time span.

3 In fact, as shown by Kohli (1983, 2004a), an improvement in the terms of trade will tend to *reduce* real GDP if it is measured by a Laspeyres quantity index. Although this remark does not apply to Canada which has moved to a superlative index to measure real GDP, most countries today still use the Laspeyres functional form, whether chained or unchained.

situation. Somehow there is the feeling that official real GDP figures do not truly reflect the income and purchasing power gains that the country has enjoyed.

> "In a special article released with yesterday's national accounts, the Statistician explained that the very strong growth in Australia's terms of trade meant the standard measures of GDP, which don't capture this, appeared to understate the true level of economic activity," Alan Wood in *The Australian*, March 8, 2005.

Not all examples are natural resources related. Switzerland, over the course of the past quarter of a century, has experienced a substantial improvement in its terms of trade — in the vicinity of 30 per cent — leading some observers to argue that the growth of real value added had been underestimated by real GDP by close to half a percentage point annually, and that the growth pessimism that has developed in that country over time was therefore not fully justified.[4]

Much of international economic theory models trade as taking place in finished products, and thus occurring *after* production. This view is somewhat misleading. Indeed, most trade is in intermediate products, and even most so-called finished goods that are traded are not ready to meet final demand. Just about all imported "finished" products must still transit through the domestic production sector where they are subject to a number of transformations (handling, transportation, insurance, repackaging, retailing, and so on). In this process, they are combined with domestic factor services, so that a significant proportion of the final price tag is typically accounted for by *domestic* value added.

Similarly, exports are not ready to meet final demand either, as they must still flow through the foreign production sector and go through a number of changes. They are thus conceptually different from goods and services intended for domestic absorption. In that sense, nearly all traded goods are intermediate goods or middle products.[5] By the same logic, all goods intended for domestic use can be viewed as nontraded goods. In truth, international trade is an intimate part of production. Trade is just another way by which some goods can be transformed into other goods. It should make little difference to economists whether products are transformed into others through a physical process, through a chemical reaction, or through trade. If the terms of this transformation become more favourable, it should not matter whether this is the result of a technological advance or whether it is due to better exchange conditions.

Although we view improvements in the terms of trade as a potentially significant source of income growth, this in no way negates the importance of the other determinants of real growth, namely increases in work effort, the accumulation of capital, improvements in total factor productivity (TFP), and net income received from abroad. Indeed, the empirical evidence suggests that in the long run growth is mostly conditioned by increases in the capital stock and by gains in TFP. Changes in the terms of trade play quantitatively a much smaller role in most cases, but for the sake of good measurement, they must nonetheless be taken into account when assessing a country's economic performance.

We should also emphasize that trade itself, beyond allowing for the transformation of some goods into others and thus being a source of welfare gains, is likely to have an indirect impact throughout much of the home economy. By exposing domestic producers to foreign competition and forcing them to continuously innovate, trade is an important catalyst of growth.

4 See Kohli (2004a, 2005), and *The Economist*, February 14, 2004.

5 See Burgess (1974) and Kohli (1978, 1991). The term "middle products" has been coined by Sanyal and Jones (1982).

This positive fallout will typically be captured by increased TFP. This article does not address this question, however, since it is fully devoted to measurement issues and it does not attempt to explain the causes of TFP growth or of terms of trade improvements. There is little doubt though that even if some of these improvements may be exogenous (the same applies to technological advances), many of them are endogenous, and the result of research and marketing activities that require the mobilization of domestic resources.

While a change in the terms of trade does have real effects, there is another price ratio, namely the price of traded vs. nontraded goods, that is prone to trigger welfare effects and that has been largely ignored in the current debate. This price ratio is often thought of as a measure of the real exchange rate.[6] An increase in the price of traded goods vs. nontraded goods (a real depreciation of the domestic currency, for given terms of trade) increases export revenues and raises the import bill. Which effect will dominate depends on the sign of the trade balance. If the trade account is in a deficit position, the latter will exceed the former, and the country will experiences a fall in its real income.[7] This outcome contrasts with the conventional wisdom that a real depreciation tends to have a positive effect on real GDP because it makes exports relatively more competitive and foreign products relatively less attractive. This is very much a Keynesian, demand side view, whereas the approach followed here is decisively neoclassical and it is fully consistent with the small open economy assumption frequently invoked in international trade theory.

These two sources of gains put together form what is sometimes called the *trading gains* — or *losses* — and they explain the difference between real GDP and real gross domestic income (GDI).[8] As shown in this article, the trading gains really consist of two separate components: the terms of trade gains and the real exchange rate gains.[9] Empirical estimates, based on the Törnqvist aggregation, show that Canada has indeed benefited handsomely from the improvement in its terms of trade that has occurred between 2002 and 2005. At the same time it has experienced a small income loss on account of real exchange rate developments.

Conventional Measures of the Trading Gains

Much of the literature on the trading gains is based on the Laspeyres aggregation, so this is a natural starting point for our analysis.[10] Real GDP is typically computed as a direct Laspeyres quantity index by simply adding up the constant dollar GDP components. National accountants

6 See Salter (1959), Dornbusch (1980), Frenkel and Mussa (1984), Corden (1992), and the literature on what has become known as the *Australian model*. Note that the real exchange rate so defined does not coincide exactly with another common definition of the real exchange rate (sometimes called the PPP real exchange rate), namely the nominal exchange rate adjusted for inflation rate differentials; see Edwards (1989) for a review of competing definitions of the real exchange rate and the Appendix for a further discussion.

7 For simplicity we abstract from factor income flows to and from abroad. Otherwise it would be the position of the current account that would be relevant, rather than that of the trade account, and one would have to focus on GNP and GNI, rather than on GDP and GDI.

8 The *trading gains* capture the income increase resulting from changes in the terms of trade and the real exchange rate between two periods of time. This is not to be confused with the *gains from trade*, which measures the welfare gains resulting from international trade starting from a closed economy situation.

9 The terms of trade and the real exchange rate are sometimes used interchangeably, but they are really two distinct concepts. Although it is true that there may be a one-to-one relationship between the two variables in the Mundell (1963)-Fleming (1962) macroeconomic model (unless tastes happen to be the same at home and abroad, in which case the real exchange rate is constant), this is generally not so if the number of goods exceeds two; see the Appendix for details.

10 See United Nations (2002), Sections 16-152-16.154, for instance. Canada currently uses the Fisher aggregation. The case of so-called superlative indexes will be examined in the next section.

have long recognized that this procedure — which involves adding exports and subtracting imports valued at their base period prices — fails to take into account the income gain or loss due to changes in the terms of trade. A number of corrective measures have therefore been proposed. They typically entail deflating nominal exports and nominal imports by the same price index. Addition of the resulting terms then yields real GDI, and the difference between real GDI and real GDP is interpreted as the trading gains.

A number of different price indexes have been proposed in the literature to deflate nominal net exports and imports. We will review the four most common choices.

- The price of imports. Deflating net exports by the price of imports amounts to retaining the purchasing power of exports — rather than their volume — when adding up the GDP components to get real GDI. This procedure is essentially the one used by the U.S. Bureau of Economic Analysis when computing Command Basis GNP, or by the Australian Bureau of Statistics when computing real GDI.[11]
- The price of exports. One can argue that one could equally well use the price of exports as the common deflator. This would mean that it is the cost of imports in terms of exports, rather than their volume, that enters the real GDI calculation.
- The price of traded goods. A third possibility, endorsed by Eurostat,[12] is to deflate the trade account by the arithmetic mean of the export and import price indexes, which can be interpreted as the price of traded goods.
- The price of domestic expenditures. This amounts to deflating the income generated by the trade balance by the price of what it will eventually be used for, i.e. to pay for domestic absorption.

Of the four proposed solutions it is the last one that we prefer, mostly for two reasons.[13] First, one can show that the GDI price deflator, which is defined as nominal GDP divided by real GDI, is then simply equal to the price of domestic expenditures.[14] Not only is this a very intuitive result, but it also means that real GDI can be obtained directly by deflating nominal GDP by the price of domestic expenditures.[15] The implicit GDI deflators for the other three proposed solutions, on the other hand, turn out to be rather complex and opaque expressions. Second, the measures of the trading gains and of real GDI are independent of the position of the trade account if the price of domestic expenditures is used, which is not true in the three other cases. In our opinion, it is important that the measure of real income be independent of the savings/absorption decision.

Nonetheless, all measures of the trading gains discussed here (including the fourth one) suffer from a number of drawbacks. Thus, they are derived in absolute (constant dollar) terms. This makes it rather difficult to assess their true importance (thus, in 2005, the Canadian trading gains amounted to $47.3 billion dollars at 1997 prices if the price of domestic expenditures is used as the deflator). Admittedly, it would be a simple matter to express them in percentage terms, relative to the current value of real GDP, for instance. More damaging though, is the fact that the very value of the trading gains reflects the choice of the base

11 See Denison (1981), and Australian Bureau of Statistics (2004).

12 See European Union (1996), Chapter 10.59.

13 The use of the price of domestic demand as the deflator is also advocated by Duguay (2006).

14 We take it, from the national accounts identity, that nominal GDP is equal to nominal GDI.

15 Yet another possibility would be to deflate nominal GDP by the price of consumption expenditures. This would introduce an additional relative price effect into the analysis, namely the one involving the price of consumption relative to total domestic expenditures; see Diewert and Lawrence (2006) for details.

period (1997 in the above example), since they are defined with reference to that period (the trading gains and losses are necessarily nil in the base period). In what follows we will therefore opt for a chained measure to get around that difficulty since the reference period is then constantly updated. Moreover, we will move to a superlative index number approach such as the one already in use in Canada.[16]

A Törnqvist Index of the Trading Gains

Canada has recently adopted the Fisher index for the measurement of real GDP. In what follows, we will use a superlative measure too, but we will opt for the Törnqvist index, rather than the Fisher index. This choice is dictated by convenience, since we find the Törnqvist aggregation more tractable. In any case, as documented by many studies, the numerical differences compared to the Fisher aggregation should be very small.[17] It should also be noted that many countries, including Canada, now use chained indexes, rather than direct (or fixed based) indexes. Our treatment will therefore also be set up in terms of chained indexes. Each chain element is defined as a growth factor, i.e. it is equal to one plus the rate of growth of the corresponding variable between period *t-1* and period *t*.

Let $V_{Y,t}$ be the growth factor of nominal GDP (thus, it is the ratio of nominal GDP at time *t* over its value one period earlier). To simplify matters we assume that all domestic GDP components can be consistently aggregated into a nontraded good, with price $P_{N,t}$. We compute $P_{N,t}$ as a Törnqvist index of the prices of consumption, investment, and government purchases. Let $P_{Y,t}$ be the GDP price deflator. It too is computed as a Törnqvist price index, of the prices of nontraded goods, imports, and exports.

We now can define $Q_{Y,t}$ as the implicit Törnqvist index of real GDP:[18]

$$(1) \qquad Q_{Y,t} \equiv \frac{V_{Y,t}}{P_{Y,t}},$$

and $Q_{Z,t}$ as the implicit Törnqvist index of real GDI:[19]

$$(2) \qquad Q_{Z,t} \equiv \frac{V_{Y,t}}{P_{N,t}}.$$

The difference between $Q_{Y,t}$ and $Q_{Z,t}$ concerns the price index that is used to deflate nominal GDP: the GDP price index in one case and the domestic expenditures price index in the other. The ratio of real GDI to real GDP defines the trading gains factor (T_t):

$$(3) \qquad T_t \equiv \frac{Q_{Z,t}}{Q_{Y,t}}.$$

T_t is greater than one if the trading gain is positive, and it is less than one if there is a trading loss. It can immediately be seen from (1)–(3) that T_t can also be obtained as:

$$(4) \qquad T_t = \frac{P_{Y,t}}{P_{N,t}}.$$

That is, the trading gains factor can be measured by comparing the two price indexes used to deflate nominal GDP. What is key here is that

16 A superlative index is an index that is exact for a flexible functional form, i.e., a functional form that provides a second-order approximation to an arbitrary aggregator function (Diewert, 1976). The main advantage of superlative indexes is that their quadratic nature enables them to better account for substitution effects that result from changes in relative prices and/or quantities. The so-called substitution bias inherent to linear index number formulas is thus reduced, or even eliminated. The best known superlative indexes are the Fisher and the Törnqvist. The Fisher index is given by the square root of a Laspeyres index and a Paasche index, which themselves can be thought of as weighted arithmetic means of the disaggregated growth factors, whereas the Tornqvist index can be viewed as a weighted geometric mean of these factors.

17 See Diewert (1978), for instance.

18 On this definition, see Diewert and Morrison (1986) and Kohli (2004b).

19 If the Fisher aggregation is preferred, one could equally well deflate nominal GDP by the Fisher price index of domestic expenditures.

the prices of imports and exports are contained in $P_{Y,t}$, but not in $P_{N,t}$.

Terms of Trade and Real Exchange Rate Effects

Let $P_{X,t}$ and $P_{M,t}$ be the price indexes of exports and imports, respectively. We define the price of traded goods ($P_{T,t}$) as the geometric mean of these two prices:

$$P_{T,t} \equiv P_{X,t}^{1/2} P_{M,t}^{1/2} \quad . \tag{5}$$

The price of traded goods in terms of nontraded goods (S_t), therefore is:

$$S_t \equiv \frac{P_{T,t}}{P_{N,t}} = \frac{P_{X,t}^{1/2} P_{M,t}^{1/2}}{P_{N,t}} \quad . \tag{6}$$

Following the Australian model literature, we will refer to this price ratio as the real exchange rate.[20] As for the terms of trade (H_t), they are defined as follows:

$$H_t \equiv \frac{P_{X,t}}{P_{M,t}} \quad . \tag{7}$$

One can show (Kohli, 2006) that T_t can be written as:

$$T_t = T_{H,t} \cdot T_{S,t}, \tag{8}$$

where

$$T_{H,t} \equiv \exp\left[\frac{1}{2}\left(\frac{\omega_{X,t-1} + \omega_{X,t}}{2} + \frac{\omega_{M,t-1} + \omega_{M,t}}{2}\right)\ln H_t\right] \tag{9}$$

captures the terms of trade effect in the Törnqvist case, and

$$T_{S,t} \equiv \exp\left[\left(\frac{\omega_{X,t-1} + \omega_{X,t}}{2} - \frac{\omega_{M,t-1} + \omega_{M,t}}{2}\right)\ln S_t\right] \tag{10}$$

is the real exchange rate effect; $\omega_{X,t}$ ($\omega_{X,t-1}$) and $\omega_{M,t}$ ($\omega_{M,t-1}$) denote the GDP shares of exports and imports at time t (t-1). These two effects measure the impact on real GDI, other things equal, of a change in the terms of trade and in the real exchange rate, respectively.

To sum up, the following decomposition of real GDI holds:

$$Q_{Z,t} = Q_{Y,t} \cdot T_{H,t} \cdot T_{S,t}. \tag{11}$$

We report in Table 1 estimates of $Q_{Y,t}$, $Q_{Z,t}$, T_t, $T_{H,t}$, and $T_{S,t}$ for Canada for the period 1982–2005.[21] Geometric means for the entire period and for selected sub-periods are reported at the bottom of the table. Focusing on the entire period first, one sees that the trading gains effect has been slightly positive, averaging about 0.1 per cent per year. This is also the difference between the average annual growth rates of real GDI (2.8 per cent) and real GDP (2.7 per cent). The trading gains effect is fully explained by the terms of trade effect; the real exchange rate effect was actually slightly negative on average. This negative contribution is explained by the fact that the price of traded goods fell relative to the price of nontraded goods over the period, which had a detrimental income effect given that Canada's trade account was in a surplus position on average. It is also apparent from Table 1 that the trading gains effect is quite volatile. It was as low as 0.9835 in 1998, thus shaving about 1.6 per cent off real GDI growth, or as high as 1.0194 in 2003, thus adding nearly 2 per cent to the growth of real income. In fact, the gain would have been even larger in 2003 had it not been for a relatively significant negative real exchange rate effect. The divergence between real GDI growth and real GDP growth in individual years can best be documented graphically. This is done in Chart 1 that shows that the discrepancies have indeed been substantial at times, particularly so in recent years.

20 See footnote 6 above and the Appendix; Corden (1992) also proposes the name "Salter ratio."

21 The data are annual and they are drawn from the Canadian national accounts (May 31, 2006 release). Thanks are due to Andrew Sharpe for providing me with the data.

Table 1
Real GDP, Real GDI, and Trading Gains for Canada

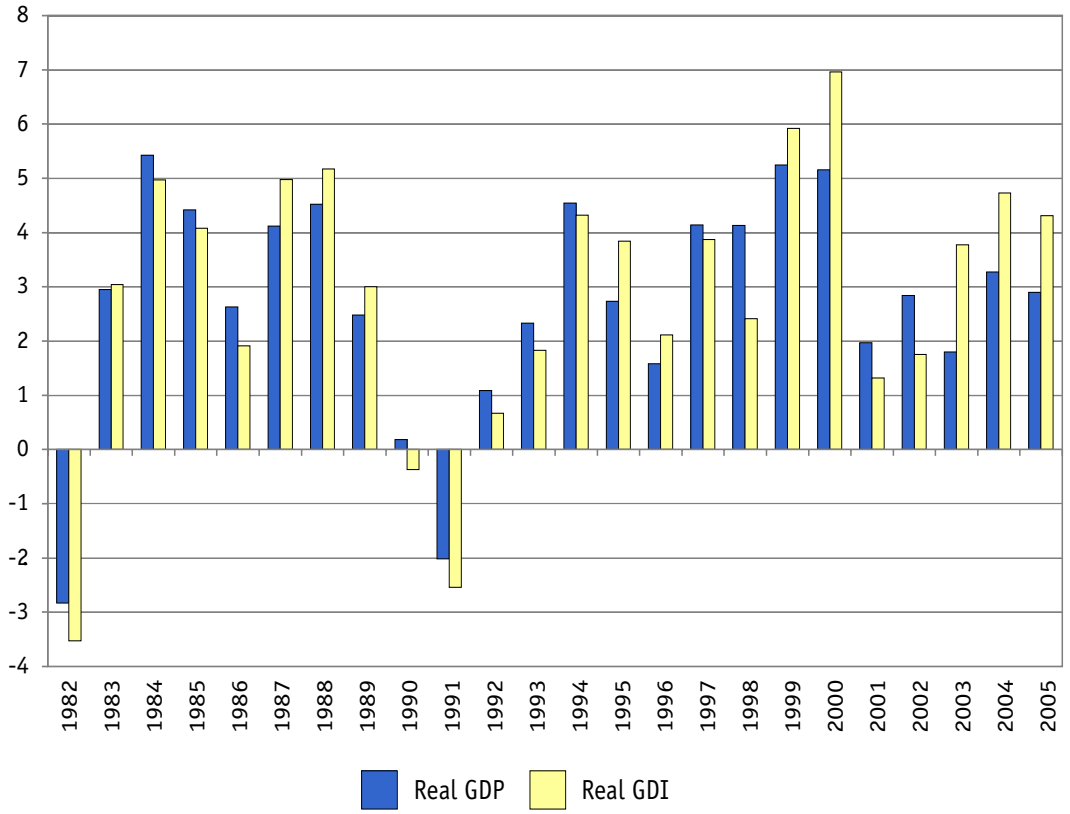
(average annual growth factors)

	Real GDP ($Q_{Y,t}$)	Real GDI ($Q_{Z,t}$)	Trading Gains (T_t)	TOT Effect ($T_{H,t}$)	ER Effect ($T_{S,t}$)
1982	0.9717	0.9647	0.9928	0.9942	0.9987
1983	1.0295	1.0304	1.0008	1.0025	0.9984
1984	1.0543	1.0497	0.9956	0.9954	1.0002
1985	1.0442	1.0408	0.9967	0.9971	0.9997
1986	1.0263	1.0191	0.9931	0.9935	0.9995
1987	1.0412	1.0498	1.0083	1.0087	0.9996
1988	1.0452	1.0517	1.0062	1.0067	0.9996
1989	1.0248	1.0300	1.0051	1.0052	0.9999
1990	1.0018	0.9963	0.9945	0.9945	1.0000
1991	0.9798	0.9746	0.9947	0.9946	1.0001
1992	1.0109	1.0067	0.9959	0.9960	0.9999
1993	1.0233	1.0183	0.9950	0.9951	0.9999
1994	1.0454	1.0432	0.9978	0.9976	1.0003
1995	1.0273	1.0384	1.0108	1.0100	1.0008
1996	1.0158	1.0211	1.0053	1.0058	0.9995
1997	1.0414	1.0387	0.9975	0.9978	0.9997
1998	1.0413	1.0241	0.9835	0.9834	1.0001
1999	1.0525	1.0592	1.0063	1.0065	0.9998
2000	1.0516	1.0696	1.0172	1.0164	1.0008
2001	1.0197	1.0132	0.9936	0.9934	1.0002
2002	1.0284	1.0175	0.9893	0.9908	0.9986
2003	1.0180	1.0377	1.0194	1.0217	0.9978
2004	1.0327	1.0473	1.0141	1.0149	0.9993
2005	1.0290	1.0431	1.0137	1.0141	0.9996
1982-1989	1.0294	1.0292	0.9998	1.0004	0.9995
1990-1999	1.0237	1.0218	0.9981	0.9981	1.0000
2000-2005	1.0298	1.0379	1.0078	1.0085	0.9994
2003-2005	1.0265	1.0427	1.0157	1.0169	0.9989
1982-2005	1.0271	1.0283	1.0011	1.0014	0.9997

Source: National Accounts, Statistics Canada, May 31, 2006.

Returning our attention to Table 1 and looking at the sub-periods, we find that the trading gains were small on average during the eighties and nineties. Since the turn of the century, however, and particularly so in the past three years, it is very clear that real GDP growth has substantially underestimated the growth in real income. Thus, from 2002 to 2005, real GDP has increased by a total of 8.2 per cent, whereas real GDI increased by 13.4 per cent. The cumulated trading gains effect of 4.8 per cent over these three years can be decomposed into a positive terms of trade effect of 5.2 per cent and a negative real exchange rate effect of 0.3 per cent. These results suggest that the view held by a number of Canadian economists, as reported by the *Financial Times* and quoted in the introduction, is indeed correct.

Chart 1
Real GDP and real GDI in Canada
(annual growth rates)



Concluding Comments

As the estimates for Canada demonstrate, growth in real value added can deviate significantly from that of real GDP. It is important that this be kept in mind, not just by economic researchers and analysts, but also by policy makers. By focusing exclusively on real GDP, policy makers might miss significant changes in the country's spending potential resulting from trading gains.

Because real GDP focuses on domestic production in a narrow sense, excluding the gains and losses generated by terms of trade and real exchange rate movements, we believe that real GDI is a better measure of real value added. Nonetheless, there are a number of other important issues that we have not covered in this article. Thus, when it comes to measuring income, one can argue that nominal gross *national* product (GNP) or income (GNI) are more relevant than GDP or GDI, given that the national concepts include net income from abroad.[22] In the case of Switzerland, for instance, given its large holdings of capital abroad, the difference is sizable, both in terms of levels and in terms of growth rates.

Another issue concerns the difference between gross and net measures of value added and income. One can certainly argue that *net* national income (NNI) is a better measure of a country's income — and absorption potential — than GNI. Erwin Diewert and his co-authors have been arguing this point very forcefully and convincingly in recent years.[23] This is particu-

22 See Kohli (2005). The same view has been expressed by Duguay (2006).

larly relevant when software expenditures are capitalized: they add substantially more to GDP (GNI) than to NDP (NNI).

What do trading gains mean for the measurement of productivity? As far as TFP is concerned, there are no consequences. Whether TFP is measured relative to real GDP or real GDI does not matter, since the difference between these two quantity indexes is fully accounted for by the trading gains. There will be a difference, however, when it comes to the *average* productivity of labour, since the numerator is not the same whether or not the trading gains are considered. Similar considerations apply to the (more relevant) *marginal* measure of labour productivity, which can be approximated by the real wage rate: its magnitude obviously depends on the price deflator that is being used. If nominal wages were deflated by the price of domestic expenditures in lieu of the GDP deflator, then both measures of labour productivity would be affected in exactly the same proportions as indicated by a comparison of (3) and (4).

One should also keep in mind that, while productivity is important, it is not the only thing that matters. As argued in this article, trading gains are very similar in nature to technological progress and increases in productivity. Moreover, the distinction between GDP and GNP (or between GDI and GNI) should remind us that if the ultimate goal of economic activities is to maximize welfare, then actions intended to increase the net income received from abroad (e.g. capital outflows to the rest of the world) make perfect sense, even though they have no impact on domestic productivity (whichever way it is measured), or on trading gains for that matter. This suggests that a fruitful direction for future research might be the development of a broader concept of productivity, i.e. a *national* measure rather than merely a *domestic* one.

Appendix: The Terms of Trade and the Real Exchange Rate

The terms of trade and the real exchange rate are sometimes used interchangeably, but they are really two distinct concepts. Although it is true that in the Mundell (1963)-Fleming (1962) model of open economy macroeconomics there can be a one-to-one relationship between the two variables, this is generally not so in models that include more than two goods.

The terms of trade and the real exchange rate in the Mundell-Fleming model

Assume two countries, the home country and the foreign country. Foreign variables are denoted by an asterisk (*). Each country produces one final good. Both goods are traded internationally. Their prices are P and P^*. As usual we assume the absence of barriers to trade and transportations costs. Our emphasis here is on real variables, so let us assume that domestic monetary policy is run in such a way that the nominal exchange rate is kept constant. Alternatively, we could assume that the currency is the same at home and abroad.

In the context of this model the terms of trade (H) are:

$$\text{(A1)} \qquad H_t \equiv \frac{P_t}{P_t^*} \ .$$

Let λ be the share of the home good in total domestic expenditures. The home country's price level (Π) can then be defined as:

$$\text{(A2)} \qquad \Pi_t \equiv P_t^{\lambda} P_t^{*1-\lambda} \ .$$

Similarly in the foreign country:

$$\text{(A3)} \qquad \Pi_t^* \equiv P_t^{\lambda^*} P_t^{*1-\lambda^*} \ .$$

The real exchange rate (E) is often defined as the ratio of the price levels of the two countries (the real exchange rate so defined is also known

23 See Diewert, Mizobuchi and Nomura (2005), and Diewert and Lawrence (2006), for instance.

as the PPP real exchange rate (Edwards, 1989)):

$$\text{(A4)} \qquad E_t \equiv \frac{\Pi_t^*}{\Pi_t} = \frac{P_t^{\lambda^*} P_t^{*1-\lambda^*}}{P_t^{\lambda} P_t^{*1-\lambda}} = H_t^{\lambda^*-\lambda} \ .$$

If preferences are such that λ is strictly larger (smaller) than λ^* E is a monotonically decreasing (increasing) function of H. For instance, if $\lambda > \lambda^*$ (i.e. if preferences are biased towards the local good in both countries) an improvement in the terms of trade (an increase in H) necessarily means a real appreciation of the home currency (a fall in E). If tastes are the same in the two countries, i.e. if $\lambda = \lambda^*$, then the real exchange rate is constant.

The terms of trade and the real exchange rate in a four-good model

Next, consider a two-country, four-good model. There are two traded, intermediate goods, and two nontraded, final goods. The home (foreign) nontraded good is labeled N (N*), and the two traded goods (adopting the home country's perspective) are identified by X and M. The price of good i ($i = N, N^*, X, M$) is denoted by P_i. This model is somewhat more general than the traditional Australian model, since we allow for two countries and we do not aggregate imports and exports into a composite good. Moreover, traded goods are treated as middle products rather than as finished goods. Note also that because there is now only one final good in each country the price of the nontraded good can be interpreted as the price level ($\Pi_t = P_{N,t}$, $\Pi_t^* = P_{N,t}^*$). The PPP real exchange rate is therefore now given by:

$$\text{(A5)} \qquad E_t \equiv \frac{P_{N,t}^*}{P_{N,t}} \ ,$$

whereas the terms of trade are given by (7) repeated here for convenience:

$$\text{(A6)} \qquad H_t \equiv \frac{P_{X,t}}{P_{M,t}} \ .$$

The ratio of traded vs. nontraded good prices is the same as (6) above:

$$\text{(A7)} \qquad S_t \equiv \frac{P_{T,t}}{P_{N,t}} = \frac{P_{X,t}^{1/2} P_{M,t}^{1/2}}{P_{N,t}} \ ,$$

and similarly in the foreign country:

$$\text{(A8)} \qquad S_t^* \equiv \frac{P_{T,t}}{P_{N^*,t}} = \frac{P_{X,t}^{1/2} P_{M,t}^{1/2}}{P_{N^*,t}} \ .$$

With four prices, there can only be three independent price ratios. Indeed, one sees that:

$$\text{(A9)} \qquad E_t = \frac{S_t}{S_t^*} \ .$$

If we assume that the home country is a small open economy, S^* can be taken as given and there is a perfect correlation between E and S, so that the two variables can be used interchangeably. E (or S) and H, on the other hand, can be viewed as fully independent variables in the small open economy case.

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