Appendix

A1 The steady-state solution of the two-sector model

Model. Lower case variables are per hour versions of inputs and outputs introduced in the text, i.e., $x_i^j = X_i^j/H^j$ is the per hour form of variable X where $i = T, N$ denotes type of good or service where relevant (i.e., ICT or other types), and $j = T, N$ denotes sector of use (ICT-producers or other producers). As in the Oulton model, the sector production functions i are Cobb-Douglas and written here in per hour form as :

$$q_N = A_N (k_T^N)^{\alpha} (k_N^N)^{\beta} (s_T^N)^{\gamma} (h_N)^{1-\alpha-\beta-\gamma} \tag{A1}$$

and

$$q_T = A_T (k_T^T)^{\alpha} (k_N^T)^{\beta} (s_T^T)^{\gamma} (h_T)^{1-\alpha-\beta-\gamma} \tag{A2}$$

The functions for the two sectors are identical except for TFP (the Hicksian shifter) whose growth rates μ_T and μ_N are exogenous.

The supply-use equations for the open economy version of the model are

$$\begin{gathered} Y = C + I + X - M \;\; ; \;\; Y = Y_T + Y_N \quad \text{where} \\ Y_T = C_T + I_T - M_C - M_I \;; \quad Y_N = C_N + I_N + X_C + X_I \;. \end{gathered} \tag{A3}$$

Imports $M = M_C + M_I$ are imports of ICT goods, and exports $X = X_C + X_I$ are exports of all other goods, i.e, the economy is an (net) importer of ICT and a (net) exporter of all other types of output. Next, we assume input supplies must equal demands, so that $H = H_N + H_T$ and $K_i = \sum_j K_i^j$, $j = T, N; i = T, N$. Accumulation equations are given by

$$\dot{K}_N = I_N - \delta_N K_N \tag{A4}$$

$$\dot{K}_T = I_T - \delta_T K_T \tag{A5}$$

Recalling that $p = P_T/P_N$, a steady state in this model is when trade is balanced $X = pM$ and when the real interest rate r and proportions of total hours allocated to each sector H_i/H $(i = T, N)$ are constant. With sectoral hours shares constant in steady state growth, for sectoral and overall output per hour to grow at a constant rate it follows that the services share of ICT production must also be constant. This follows from the definitions:

$$Q_T \equiv Y_T + S_T^N \;\; \text{and} \;\; Q_N \equiv Y_N \tag{A6}$$

where in the steady state, the growth rate Q_N and Y_N (and thus q_n and y_n) are identical by definition. The growth rate of Q_T is a (constant) share-weighted average of the growth rates of Y_T and S_T^N, which grow at the same rate, and thus imply $\dot{q}_T = \dot{y}_T$ in steady growth.

Note also that with sectoral hours shares constant in steady state growth, output-per-hour (OPH) growth in the total economy is a share-weighted average of the growth rates of OPH growth in each of the sectors, i.e., accounting for "labor reallocation" due to shifts in hours shares is not needed.

Growth rate of relative ICT prices ($\dot{p}$). Given the model's assumption that production functions are the same up to a scalar multiple, it is easy to see that the rate of change in relative ICT prices $\dot{p}$ (where recall $p = \frac{P_T}{P_N}$) is given by

$$\dot{p} = \mu_N - \mu_T < 0 \tag{A7}$$

Equation A7 is proved by total differentiation of the payments equations, text equation (4), with respect to time. With μ_N and μ_T constant by assumption, so too is $\dot{p}$.

Growth rate of output per hour. To obtain the steady state growth rates of labor productivity, first differentiate equation (A1) and (A2) with respect to time, which gives

$$\dot{q}_N = \mu_N + \alpha \dot{k}_N^N + \beta \dot{k}_T^N + \gamma \dot{s}_T^N + (1 - \alpha - \beta - \gamma)\dot{h} \tag{A8}$$

$$\dot{q}_T = \mu_T + \alpha \dot{k}_N^T + \beta \dot{k}_T^T + \gamma \dot{s}_T^T + (1 - \alpha - \beta - \gamma)\dot{h} \tag{A9}$$

where from (A6) we have

$$\dot{q}_N = \dot{y}_N \quad \text{and} \quad \dot{q}_T = \dot{y}_T \quad . \tag{A10}$$

Consider first the N sector. Profit maximization requires that the real user cost equals the real marginal product of capital, which for nonICT and ICT capital are given by

$$(i + \delta_N) = \alpha \frac{q_N}{k_N^N} \quad \text{and} \quad (i + \delta_T - \dot{p})p = \beta \frac{q_N}{k_T^N} \quad . \tag{A11}$$

where i is the nominal rate of interest and the real interest rate is the nominal rate minus the growth rate of the N sector price P_N, expressed in terms of the relative price p in (A11).

In steady state where the real interest rate is constant and factors are paid their marginal products, the solutions for sector N are then

$$\dot{q}_N^* = \dot{y}_N^* = \dot{k}_N^{N*} \tag{A12}$$

$$\dot{q}_N^* = \dot{y}_N^* = \dot{k}_T^{N*} + \dot{p} \tag{A13}$$

where * denotes a steady state solution (recall $\dot{p}$ is constant by assumption).

Consider now the T sector. Equality of the real marginal product of ICT capital in T sector production with real user cost implies

$$(i + \delta_T - \dot{p}) = \beta \frac{q_T}{k_T^T} \tag{A14}$$

Because the left hand side of (A14) is constant, it follows that $\dot{q}_T = \dot{k}_T^T$ from which it follows:

$$\dot{q}_T^* = \dot{y}_N^* - \dot{p} \quad . \tag{A15}$$

In steady state growth, output per hour in sector T grows faster than output per hour in sector N.

Finally, equality of the real marginal product of ICT intermediate services across the two sectors implies that

$$\frac{\partial q_N}{\partial s_T^N} = \gamma \frac{q_N}{s_T^N} = \gamma \frac{Y_N}{S_T^N} \tag{A16}$$

must be identical to

$$\frac{\partial q_T}{\partial s_T^T} = \gamma \frac{q_T}{s_T^T} = \gamma \frac{Y_T + S_T^N}{S_T^T} \tag{A17}$$

Equation (A16) implies that $\dot{s}_T^N$ is equal to $\dot{y}_N$ in steady state growth but from (A15), we know that q_T, of which s_T^N is a component, grows at a faster rate than $\dot{y}_N$. It is readily seen that the condition $\dot{q}_T = \dot{y}_T$ solves this dilemna and implies

$$\dot{s}_T^N = \dot{y_N} - \dot{p} \tag{A18}$$

$$\dot{y}_T = \dot{y}_N^* - \dot{p} \ . \tag{A19}$$

Now substitute equations (A12), (A13), and (A18) into (A8), the expression for growth in output per hour in sector N:

$$\dot{y}_N = \mu_N + \alpha\dot{y}_N + \beta(\dot{y}_N - \dot{p}) + \gamma(\dot{y}_N - \dot{p}) + (1 - \alpha - \beta - \gamma)\dot{h}$$

which after rearranging terms yields

$$\dot{y}_N = \frac{\mu_N - (\beta + \gamma)\dot{p}}{(1 - \alpha - \beta - \gamma)} + \dot{h} \ . \tag{A20}$$

Define the steady state output share of the T sector as

$$\omega_T^* = \frac{pY_T}{Y_N + pY_T} \tag{A21}$$

in which case the steady state OPH growth rate for the total economy may be written as

$$\begin{aligned} \dot{y}^* &= (1 - \omega_T^*)\dot{y}_N^* + \omega_T \dot{y}_T^* \\ &= \dot{y}_N^* + \omega_T^*(\dot{y}_T^* - \dot{y}_N^*) \ . \end{aligned} \tag{A22}$$

Substituting (A19) into (A22) yields

$$\dot{y}^* = \dot{y}_N^* - \omega_T^* \dot{p}$$

and subsitituting (A20) into this expression and combining terms yields our final result, an expression for the contribution of the ICT sector to total OPH growth:

$$\begin{aligned} \dot{y}^* &= \frac{\mu_N - (\beta + \gamma)\dot{p}}{(1 - \alpha - \beta - \gamma)} + \dot{h} - \omega_T^* \dot{p} \\ &= \frac{\mu_N}{(1 - \alpha - \beta - \gamma)} + \dot{h} \ + \ \underbrace{\frac{(\beta + \gamma)(-\dot{p})}{(1 - \alpha - \beta - \gamma)} + \omega_T^*(-\dot{p})}_{\textit{Contribution of ICT to total OPH growth}} \end{aligned} \tag{A23}$$

The final term in equation (A23) appears as text equation (7) where $\beta, \gamma, (1 - \alpha - \beta - \gamma)$, and ω_T are replaced by their empirical counterparts $\overline{v}_{K_T}, \overline{\zeta}_T^N, \overline{v}_L$, and $\overline{w}_T$.

Contribution of ICT to growth in TFP The amended model's solution for aggregate TFP μ also is different than that implied by the original Oulton model. Under the usual neoclassical growth accounting assumptions in the presence of intermediates (e.g., Hulten 1978), the growth of aggregate

TFP is the sum of the growth of each sector's TFP growth times its Domar-Hulten weight, which is the ratio of each sector's sectoral production (gross output net of own use) to aggregate value added, $P_i Q_i / PY$. From (2) and (5), these weights are expressed as:

$$\frac{P_T Q_T}{PY} = \overline{w}_T + \overline{\zeta}_T^N \quad ; \quad \frac{P_N Q_N}{PY} = 1 - \overline{w}_T$$

whose sum is greater than one by the relative size of ICT services supplied to nonICT producers.

The growth of aggregate TFP μ is then given by

$$\text{(A24)} \qquad \mu = \underbrace{(\overline{w}_T + \overline{\zeta}_T^N)\mu_T}_{\textit{Contribution of ICT sector}} + (1 - \overline{w}_T)\mu_N$$

The contribution of the ICT sector to overall TFP growth is larger than the sector's share in final demand $\overline{w}_T$ to account for the diffusion of the sector's innovation via use of ICT services (intermediate inputs) by other producers in the economy.

A2 Nominal ICT investment deflators

The table below shows detailed components of the nominal national accounts-style price deflators calculated for the analysis in this paper. The methods used to construct these deflators are described in detail in our companion paper (Byrne and Corrado, 2017b).

Table A1: **Nominal ICT Investment Price Change (annual rate)**

		1963 to 1987 (1)	1987 to 2004 (2)	2004 to 2015 (3)	1994 to 2004 (4)	2004 to 2008 (5)	2008 to 2015 (6)
1.	**ICT investment**	-4.9	-10.6	-8.0	-12.4	-8.9	-7.5
2.	Communication equipment	.4	-7.3	-8.7	-9.1	-7.4	-9.5
3.	Telecom	-.3	-11.7	-12.4	-14.3	-10.1	-13.7
4.	Other equipment	.4	-8.3	-9.3	-10.3	-8.1	-10.0
5.	Capitalized services	–	1.1	-3.7	-.1	-2.5	-4.3
6.	Computers and peripherals	-17.1	-21.2	-17.0	-24.0	-21.8	-14.1
7.	Servers and storage	-18.1	-25.2	-25.7	-31.0	-30.6	-22.7
8.	PCs	–	-27.9	-23.4	-30.3	-30.2	-19.2
9.	Other equipment	-9.0	-9.3	-3.3	-8.8	-5.4	-2.0
10.	Capitalized services	–	-2.0	-2.2	-3.1	-1.5	-2.6
11.	Software	-1.0	-4.4	-3.9	-5.5	-3.5	-4.1
12.	Prepackaged	-9.8	-9.0	-7.0	-9.6	-6.8	-7.2
13.	Custom and own-account	.0	-2.0	-2.2	-3.1	-1.5	-2.6
Memos:							
14.	ICT excluding PCs	-4.5	-8.4	-6.5	-9.9	-6.6	-6.4
15.	Computers excluding PCs	-16.6	-17.1	-11.6	-19.8	-14.5	-9.9
16.	BEA ICT	-2.7	-6.4	-2.1	-7.5	-3.3	-1.4
17.	BEA ICT excluding PCs	-2.6	-4.5	-1.4	-5.2	-1.9	-1.2
18.	Computers excluding PCs	-16.6	-11.0	-3.6	-12.7	-6.6	-1.8

NOTE: Figures reported as "BEA" are authors' calculations based on BEA data.