Industry Productivity Performance in G7 Countries after the Global Financial Crisis: A Canadian Perspective

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Abstract

Compared to other G7 countries, Canada performed relatively well after the global financial crisis in both aggregate labour productivity and multifactor productivity (MFP) growth, which is in sharp contrast to its relatively weak performance in the 2000s. The objective of this article is to analyse the sources of Canada's superior performance by comparing it to other G7 countries. In particular, the article examines industry productivity performance, estimates the contribution of individual industries to both aggregate labour productivity and MFP growth, and highlights the differences between Canada and other G7 countries before and after 2010. To this end, this article develops a methodology to more precisely identify the underlying forces at the industry level that have either propelled or hindered productivity growth, and thereby identify which industries have been at the heart of the changes in productivity performance.

Canada's weaker productivity growth in the post-2000 period and compared to other G7 countries, especially the United States, has been subject to an extensive research (e.g. Tang, 2014 and 2017, and more recently Gu and Willox, 2018, and Sharpe and Tsang, 2018). Canada underperformed most of the OECD countries in productivity growth in the first decade after 2000. Among G7 countries, Canada's average annual aggregate labour productivity (real GDP per hour worked) growth rate over the periods of 2000-2010 was the second lowest at 1.07 per cent per year, only

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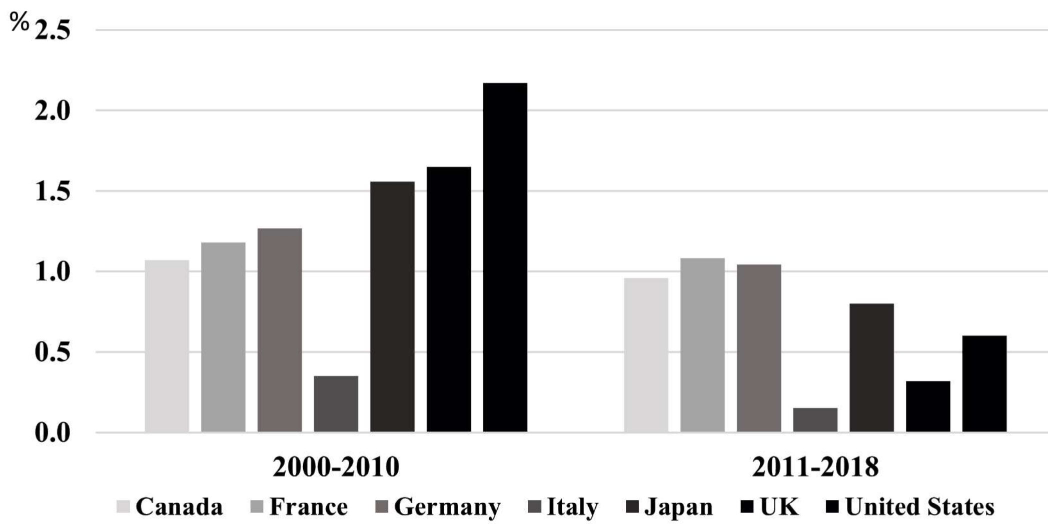
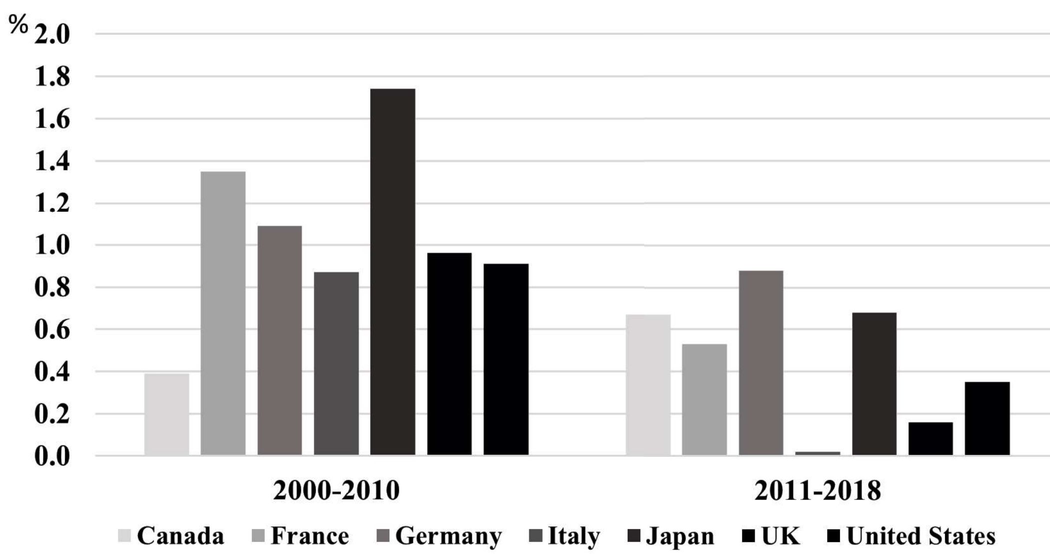
Chart 1: Total Economy Productivity Growth in G7 Countries, 2000-2010 and 2011-2018 (per cent per year)

Panel A: Labour Productivity



Panel B: Multifactor Productivity



Source: OECD Productivity Data

ahead of Italy, and significantly lower than that for Japan (1.56 per cent), the United Kingdom (1.65 per cent) and the United States (2.17 per cent) (Chart 1, Panel A).[2] Also, Canada's average aggregate multifactor productivity (MFP) growth rate over the same period was the lowest and lagged significantly behind the other G7 countries (Chart 1, Panel B).

After recovering from the global financial crisis, however, Canada has emerged to become one of the leaders in produc-

2 The growth rate for a period is the average of the annual growth rates over the period.

3 Note that Canada's relatively better productivity growth is sensitive to the period in question. For example, Canada was not one of the best productivity growth performers among G7 countries in 2015-2018. Also note that in this article we focus on productivity growth, not productivity levels. In terms of the total economy productivity level (output per hour), Canada still lags behind five G7 countries, only ahead of Japan.

tivity growth among G7 countries, at least up to 2018.[3] Canada was the third highest in labour productivity growth and the second highest in MFP growth (Chart 1). Over the 2011-2018 period, Canada experienced a 0.11 percentage points reduction in total economy labour productivity growth compared to the rate in 2000-2010, and the reduction was the second smallest after France (-0.10 percentage points). Canada was the only country that accelerated its MFP growth in 2011-2018 relative to 2000-2010 (0.28 percentage points), compared to MFP growth falls in other G7 countries ranging from -0.21 percentage points (Germany) to -1.06 percentage points (Japan).

Thus, Canada's improvement in its relative position in productivity growth performance among G7 countries between those two periods was on one hand due to Canada's persistent ability to maintain labour productivity growth or improve MFP growth after the global financial crisis. On the other hand, it was due to the slowdown in both labour productivity and MFP growth in other G7 countries. The decline in the United States, the leader of the labour productivity growth over the 2000-2010, was especially compelling. Labour productivity and MFP growth rates in the United States decreased from 2.17 and 0.91 per cent per year in 2000-2010 to 0.60 and 0.35 per cent per year in 2011-2018, respectively.

Productivity advance is the key for economic growth and for the improvement in living standards. There are also other advantages for a country to improve its productivity relative to other countries. As higher productivity growth in a country raises the real income of the country's citizens at a faster pace than those in other countries, it helps to attract and retain capital and skilled labour in the country as factors of production become increasingly footloose in the globalized economy. This can create a virtuous cycle for productivity and economic growth.

For Canadian policy development to further improve productivity, it is important to better understand the differences of the sources or components of productivity growth between Canada and other countries.

The objective of this article is to examine what are the driving forces for Canada's relatively better productivity growth performance after 2010 with a comparison to other G7 countries.[4] To this end, building on our earlier research, we use a methodology that is able to more precisely identify the industries and the underlying forces or components that have either propelled or hindered productivity growth, and to observe which industries have been at the heart of the changes in productivity performance.[5]

Our methodology requires data for the G7 economies at the industry level. To ensure comparability, such industry-level

4 We choose 2010 as the break year since most of the countries had re-established their economies following the crisis only after 2010. Note, however, that when 2009 is used, Canada still improved substantially its productivity growth position among G7 countries in 2009-2018 (or 2010-2018) compared to in 2000-2008.

5 Note that this article only studies industry performance and contributions to aggregate productivity growth. It does not attempt to examine factors driving industry performance. For such studies, see Sharpe and Tsang (2018), Gu (2019), and Tang *et al.* (2020).

data need to be developed consistently. Thus, in this article, we use KLEMS data, which will be discussed in detail later on. At the time of writing, the latest year for which data were available for all G7 countries was 2015, and thus our analysis at the industry level must end in 2015. Using the data, we estimate the contribution and the components of these contributions of individual industry to both aggregate labour productivity and MFP growth and highlight the differences that have emerged between Canada and other G7 countries in 2000-2010 and 2011-2015.

The findings in this article will contribute to a better understanding of Canada's relatively improved position in productivity growth performance after the global financial crisis compared to the other G7 countries.

This article is divided into four main sections. Section one explains the methodology used to decompose labour productivity and multifactor productivity into three components, within-industry productivity effects, price effects and size effects. Section two presents the empirical results for G7 countries for the 2000-2010 and 2011-2015 periods. Section three discusses the results in more depth for three industries: mining and quarrying, and manufacturing. The fourth and final section concludes.

Methodology

The aggregate productivity performance of an economy depends on productivity growth by industry and changes in industrial structure as well. In Appendix A, we set up an analytical framework to trace a change in aggregate productivity growth to the corresponding changes at an industry level. We build on the decomposition method originally developed by Tang and Wang (2004), which has been followed up by Diewert (2015), Tang and Wang (2015), and Zhao and Tang (2015).[6]

For both labour productivity and MFP, output at the aggregate level is value added while at the industry level, it is gross output. This means that to estimate industry contribution to aggregate productivity, we have to "peel" off the effect of intermediate inputs.[7]

Decomposing Aggregate Labour Productivity Growth

In the Appendix, we show that an industry affects the aggregate labour productivity level through three channels. The first two channels are the industry own labour productivity level and the relative gross output price, which are partly offset by intermediate input intensity and the relative intermediate input price. The third channel is the size of the industry in the economy, which is indicated by the industry's share of total hours worked. This is consistent with the literature where the labour share is often used for aggregating labour productivity from the industry level.

A higher relative price of an industry's

6 A theoretical and empirical justification of the Tang-Wang method is provided by Dumagan (2013).

7 When output at the industry level is also value added, the framework will be the same but without the terms associated with intermediate inputs. It is much simpler.

gross output, after the offsetting effect of the relative price of intermediate goods, positively contributes to aggregate labour productivity.[8] Equally, a lower relative price of an industry's gross output contributes negatively to aggregate productivity. Prices represent the ability of the industry to generate economic value in the economy. This is consistent with the chained-Fisher index in aggregation, which gives a greater weight to the industries with higher output prices. The chain-Fisher index has been adopted by many statistics agencies around world for real GDP estimation, including those in G7 countries.[9]

In terms of growth, we show that an industry can contribute positively to aggregate labour productivity growth through three channels: an improvement in its gross output labour productivity, a rise in its relative gross output price, and an increase in its share of total hours worked. This can be partly offset by the effect of the changes associated with intermediate input intensity, relative intermediate input prices and the labour input share.The "peeling-off" of the effects from intermediate inputs is necessary as output at the aggregate level is in value added while it is gross output at the industry level.

The contributions of the industry associated with gross output are weighted by the ratio of the industry's nominal gross output to nominal GDP in the period. Similarly, the effects associated with intermediate inputs are weighted by the ratio of the industry's nominal intermediate inputs to nominal GDP at the initial period. The weights are analogous to the "Domar weights" used to aggregate industry multifactor productivity growth calculated based on the gross output concept. Domar weights were originated by Domar (1961). They are the ratios of industry's current dollar gross output divided by aggregate value added in current dollars. The weights sum to more than one, reflecting the integration across industries. They have been widely used to aggregate gross output-based productivity at the industry level to value added-based productivity at the business sector or the economy level.

There are four components in the decomposing of aggregate labour productivity growth. The first component is called the productivity effect, or the within-industry productivity effect, as it is an industry contribution to aggregate labour productivity growth from the industry's labour productivity improvement after controlling for the

8 The price effect is the effect from changes in the industry price relative to the aggregate price. It is not necessarily zero for three reasons. First, the price effect is a weighted sum. Second, the sum of the change in industry relative price is not by definition equal to zero. According to our formulation in the Appendix, the aggregate price is a value added price while industry price is a gross output price. The aggregate price is implicitly a chain-Fisher index of industry value added prices. Third, the price effect is also complicated by "peeling off" the effect from the change in the relative intermediate input price.

9 There is no standard approach to decompose aggregate productivity growth. Some economists argue that there should be no price effect related to industry contributions to aggregate productivity growth (e.g. Reinsdorf, 2015).They point out that the inclusion of relative prices effects in industry contribution to aggregate productivity growth, even though they are offsetting, can result in industries with large increases in prices making a large contribution to aggregate productivity growth, even though within-industry productivity growth may have been negative as in the oil and gas industry. Equally, industries with very large decreases in relative prices caused by very large within-industry productivity gains, can make a negative contribution to aggregate productivity growth.

effect from the change in the intermediate input intensity of the industry. Similarly, the second component is called the price effect as it is associated with the improvement in the relative gross output price of the industry after controlling for the effect from a change in its real intermediate input price. The effect captures the industry's ability in creating economic value. The third component is called the size effect as for aggregate labour productivity, the labour share indicates the industry size relative to other industries. The last component captures the interactions between the first three components. It tends to be small as it consists of second-order effects.

The framework used in this article is an extension of Tang and Wang (2004, 2015) as the decomposition here separates the effects from changes in relative prices and hours shares. It is an extension of Diewert (2015) from a value added framework to a gross output framework.

Decomposing Aggregate MFP Growth

Labour productivity is only a partial indicator of productivity, which can be influenced by capital intensity and intermediate input intensity. To study the overall productivity performance associated with labour, capital, and intermediate inputs, the analytical framework for labour productivity growth by Tang and Wang (2004, 2015) can also be modified and extended for studying MFP growth.

As in labour productivity, the formulation in the Appendix shows that an industry can contribute to aggregate MFP level through three channels. The first two channels are industry own MFP performance and the ratio of industry relative output prices to industry relative input prices, which is partly offset by the industry's intermediate input intensity (in all inputs) and the ratio of industry intermediate input prices to industry total input prices. The third channel is the relative size of the industry in the economy, which is indicated by the ratio of industry total input cost to aggregate total input cost. This is equivalent to the ratio of industry nominal gross output to aggregate GDP since nominal gross output equals total input cost at the industry level and nominal value added (or GDP) equals total aggregate total input cost at the aggregate level.

The format of the formulation for industry effects on aggregate MFP growth is the same as that for aggregate labour productivity growth. An industry can contribute to aggregate MFP growth by improving its own MFP, raising its output price relative to the total input price, and increasing its relative size in terms of nominal output. The contributions are partly offset by the corresponding effects associated with intermediate input use. As in labour productivity, here we designate the first component the productivity effect, or the within-industry productivity effect. This is the industry's contribution to aggregate MFP growth from its MFP improvement after controlling for any change in intermediate input intensity. Similarly, we designate the second component as the price effect as it is associated with the improvement in the relative gross output price of the industry after controlling for any change in its relative intermediate input prices. The third component is called the size effect as for aggregate MFP, the ratio of total industry input cost to total aggregate input cost indicates the industry size relative to other industries. The last component captures the interactions between the first three components. It tends to be small as it consists of second-order effects.

Again, the decomposition framework for MFP is an extension of Diewert (2015) from industry output in value added terms to industry output in gross output terms.

Empirical Results

Using the framework outlined above, we now estimate the industry contribution to aggregate productivity growth in Canada, with a comparison to other G7 countries. We first describe the data sources and then discuss the empirical results. We highlight the industries that either propelled or hindered aggregate productivity growth.

Data

The application of our methodology for the estimation of industry contributions and the contribution components to aggregate productivity growth in an economy requires output data at the industry level. To this end, we extract the necessary data for our analysis from Statistics Canada for Canada, from the Institute of Economic Research at Hitotsubashi University for Japan, and from either the EU KLEMS or the World KLEMS databases for France, Germany, Italy, the United Kingdom and the United States. These databases are made publicly available through the World KLEMS initiative based on Harvard University, and were supported by the EU KLEMS program based at the Groningen Growth and Development Center at the University of Groningen.[10] The databases are fairly comparable and consistently developed. They allow for reliable comparisons in industrial structure and sources of productivity growth among G7 countries.

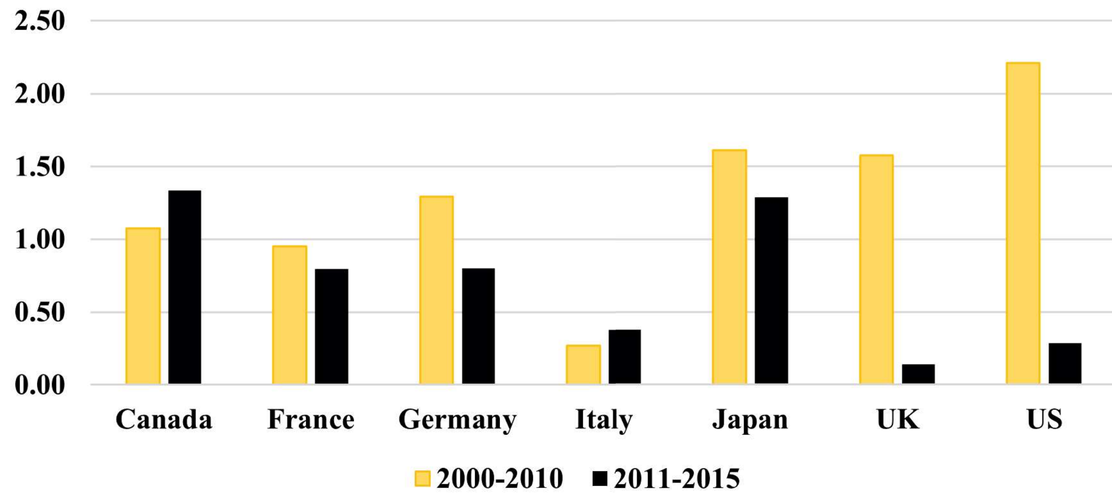
For Canada, the KLEMS data only cover the business sector. To ensure comparability, for other G7 countries, we exclude public administration, defence, compulsory social security, activities of households as employers, and activities of extraterritorial organizations from output in other G7 countries.

For all countries, we combine industries into 12 major industry groups: agriculture, forestry and fishing; mining and quarrying; manufacturing; utilities; construction; wholesale and retail trade; transportation and storage; accommodation and food services; information and communication; financial, insurance and real estate activities (FIRE); professional, scientific, technical, administrative and support service activities (PSTAS); and community social and personal services. We call the aggregate of those 12 industries "the total business sector". Real output and inputs for the business sector are aggregated from the industry level using the Törnqvist quantity in-

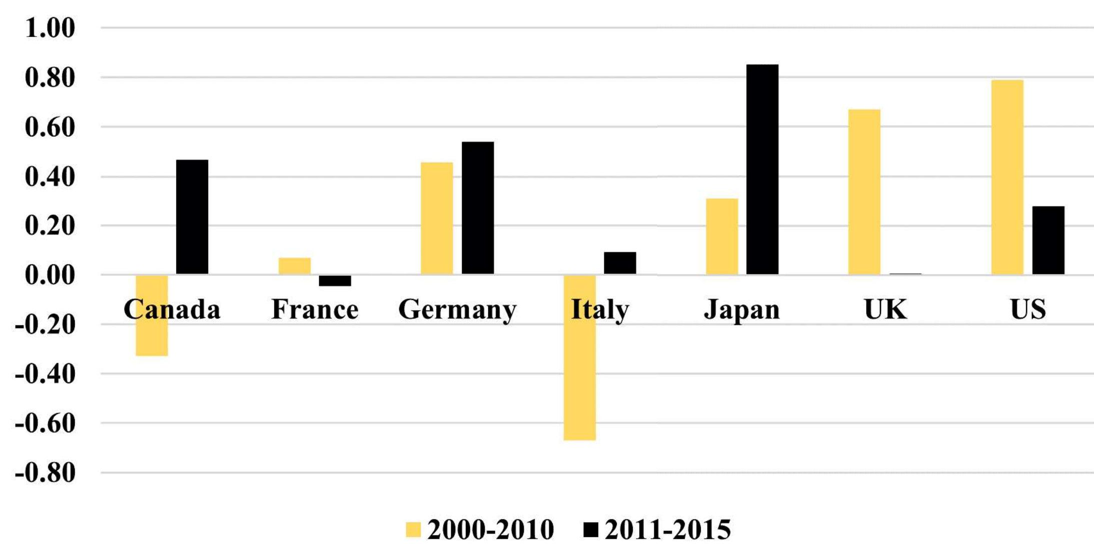
10 The KLEMS database for the United Kingdom has no price data for intermediate inputs as, unlike other national statistical offices which use double deflation to estimate real output at the industry level, the UK statistical office uses a single-deflation approach in its national accounts. As a proxy for the missing data for the United Kingdom, we use the average of intermediate input prices in other G7 countries in Europe (France, Germany and Italy). The proxy is expected to be reasonable as our analysis is only concerned with changes in intermediate input price, not its level.

Chart 2: Business Sector Productivity Growth in G7 Countries, 2000-2010 and 2011-2015 (per cent per year)

Panel A: Labour Productivity



Panel B: Multifactor Productivity



Source: Authors' calculation based on KLEMS data

dex, which is almost the same as the chain-Fisher index.

For all countries, we have data up to 2015. We focus our analysis on 2000-2015, which is divided into two sub-periods 2000-2010 and 2011-2015. The division point was chosen to compare productivity performance among those countries after the global financial crisis. Most of economies started to recover significantly in 2011.

Empirical Results on Aggregate Productivity Growth in the Business Sector

Before we discuss the empirical results based on KLEMS data, it should be noted that the productivity estimates for the G7 countries in Chart 1 from the OECD are for total economy and also the estimates of labour and capital may be different from KLEMS data in terms of controlling for

Table 1: Components in the Change of Business Sector Productivity Growth between 2000-2010 and 2011-2015 in G7 Countries (percentage points per year)

Panel A: Labour Productivity							
	Canada	France	Germany	Italy	Japan	UK	United States
Industry Productivity	0.79	-0.10	-0.48	0.24	0.49	-1.70	-2.13
Price Effect	-0.01	-0.12	0.01	-0.09	-0.57	0.50	-0.02
Size Effect	-0.51	0.05	-0.02	-0.07	-0.30	-0.21	0.15
Interactions	0.00	0.02	-0.01	0.03	0.06	-0.02	0.08
Total	0.26	-0.15	-0.49	0.11	-0.32	-1.43	-1.93
Panel B: Multifactor Productivity							
	Canada	France	Germany	Italy	Japan	UK	United States
Industry Productivity	0.90	0.40	0.81	0.95	1.01	-0.41	-1.07
Price Effect	-0.19	-0.03	0.52	0.57	0.18	0.31	0.28
Size Effect	0.01	-0.52	-1.28	-0.79	-0.65	-0.58	0.14
Interactions	0.07	0.03	0.03	0.04	0.01	0.02	0.15
Total	0.80	-0.12	0.08	0.76	0.55	-0.66	-0.51

Source:Authors' calculation based on KLEMS data.

quality in labour and capital by considering the changes in their compositions. The time periods for the two charts are also different. As a result, the productivity estimates for the total economy from OECD are different from those for the business sector based on KLEMS (Chart 2).

However, the general message from these two productivity series is consistent, that is, Canada improved its productivity growth significantly relative to other G7 countries after the global financial crisis. Labour productivity growth in the Canadian business sector increased from an average of 1.07 per cent per year in 2000-2010 to 1.34 per cent per year in 2011-2015, an increase of 0.26 percentage points per year. The improvement was even larger in MFP from -0.33 per cent per year to 0.47 per cent per year, an increase of 0.80 percentage points.

The improvement in labour and multifactor productivity in Canada was the largest among the G7 economies. This was followed by Italy which also saw a significant improvement in both labour productivity (0.11 percentage points) and MFP (0.76 percentage points), and Japan in MFP (0.55 percentage points). In contrast, all other G7 countries experienced a decline in labour productivity growth between periods. The largest decline was in the United States. Its labour productivity growth rate declined from 2.21 per cent per year in 2000-2010 to only 0.28 per cent per year in 2011-2015. Similarly, its MFP growth rate also dropped from 0.79 per cent per year to 0.28 per cent per year.

In an on-line appendix to this article,[11] we report and discuss each country's detailed industry contribution to both aggregate labour productivity growth and aggregate MFP growth, which are estimated using the framework developed in the Ap-

11 The on-line appendix is available at www.csls/ipm39/Tang_Wang_Appendix.

pendix. In the remainder of this section, we focus on the comparison of industry contribution to aggregate labour productivity growth and aggregate MFP growth in Canada relative to the other G7 countries.

Country Comparison: Industry Performance and Aggregate Labour Productivity Growth

Now we compare Canada to other G7 countries and highlight the forces behind Canada's better labour productivity growth after recovering from the global financial crisis. We focus in this subsection on the change in labour productivity growth between the two sub-periods between 2000-2010 and 2011-2015.

Panel A of Table 1 provides a country comparison of the components of the change in labour productivity growth between 2000-2010 and 2011-2015 at the business sector level. It shows that Canada's acceleration in labour productivity growth of 0.26 percentage points after the global financial crisis was the largest among the G7 countries. The superior performance is explained by the large increase in within-industry productivity at 0.79 percentage points, the largest in the G7. Canada also experienced the largest decline in the size effect, but the drop in the size effect only partially offset the increase in the productivity effect. In contrast, the United States had the largest drop in labour productivity growth. This was mainly because the country experienced the largest drop in the labour productivity effect.

Compared with other G7 countries, the positive change in industry contributions to the accelaration in labour productivity growth in Canada was widespread across industries (Table 2, Panel A). In particular, Canada led in industry contributions from agriculture, forestry and fishing; manufacturing, transportation and storage; and FIRE. Due to the strength of its financial sector and housing market, Canada was the only country experiencing a positive increase in industry contribution from FIRE. Notably, Canada also had the largest drop in the industry contribution from mining and quarrying. This was because the Canadian economy is more resource-based. Oil and other commodity prices substantially dropped starting in 2014.

In contrast, the United States had the largest negative contributions to aggregate labour productivity growth in half of the 12 industries: agriculture, forestry and fishing; manufacturing; wholesale and retail trade; FIRE; PSTAS; and community, social and personal services.

In sum, Canada's greater acceleration in business sector labour productivity growth than other G7 countries was largely due to the slower decline in the size of manufacturing and the strength of FIRE.

Country Comparison: Industry Performance and Aggregate MFP Growth

Canada's acceleration in MFP growth between the two sub-periods was also the largest among G7 countries, followed closely by Italy and Japan. On the other hand, the United States experienced the largest decline in MFP growth during the same periods.

Panel B of Table 1 shows that Canada's superior performance in MFP was mainly due to a significant increase in the produc-

Table 2: Industry Contribution to the Change in Business Sector Productivity Growth between 2000-2010 and 2011-2015 (Percentage points per year)

Panel A: Labour Productivity							
	Canada	France	Germany	Italy	Japan	UK	US
Agriculture, forestry and fishing	0.21	0.06	0.00	0.16	0.03	-0.01	-0.06
Mining and quarrying	-1.12	0.00	-0.01	0.03	0.01	-0.32	-0.28
Manufacturing	0.90	0.45	0.01	0.36	-0.15	0.24	-0.08
Utilities	0.07	0.12	-0.18	-0.02	0.00	0.02	-0.11
Construction	-0.05	-0.26	0.14	-0.28	0.41	-0.04	0.06
Wholesale and retail trade	-0.07	-0.06	-0.06	0.22	-0.10	-0.23	-0.13
Transportation and storage	0.18	-0.02	-0.12	0.01	-0.03	0.05	-0.03
Accommodation and food services	0.07	-0.01	0.03	0.01	0.04	0.08	-0.01
Information and communication	-0.06	-0.03	0.05	-0.19	-0.14	-0.04	-0.18
FIRE	0.13	-0.24	-0.32	-0.06	-0.25	-0.30	-0.49
PSTAS	0.01	-0.08	0.02	-0.03	0.00	-0.09	-0.18
Community social and personal services	-0.01	-0.09	-0.06	-0.11	-0.16	-0.79	-0.43
Total Business Sector	0.26	-0.15	-0.49	0.11	-0.32	-1.43	-1.93
Panel B: Multifactor Productivity							
	Canada	France	Germany	Italy	Japan	UK	US
Agriculture, forestry and fishing	0.22	0.06	0.01	0.18	0.05	0.00	-0.05
Mining and quarrying	-1.08	0.00	-0.01	0.04	0.01	-0.31	-0.25
Manufacturing	1.05	0.48	0.14	0.49	0.09	0.34	0.14
Utilities	0.09	0.12	-0.16	0.00	0.04	0.03	-0.08
Construction	-0.03	-0.26	0.17	-0.24	0.48	0.01	0.13
Wholesale and retail trade	0.01	-0.05	0.00	0.31	0.05	-0.13	0.07
Transportation and storage	0.21	-0.02	-0.09	0.05	0.02	0.09	0.02
Accommodation and food services	0.08	-0.01	0.04	0.03	0.07	0.10	0.04
Information and communication	-0.03	-0.03	0.08	-0.15	-0.08	-0.01	-0.08
FIRE	0.21	-0.25	-0.22	0.06	-0.15	-0.15	-0.18
PSTAS	0.05	-0.08	0.09	0.03	0.01	0.00	-0.03
Community social and personal services	0.02	-0.09	0.03	-0.02	-0.05	-0.66	-0.25
Total Business Sector	0.80	-0.12	0.08	0.76	0.55	-0.66	-0.51

Note: FIRE stands for financial, insurance and real estate activities; PSTAS stands for professional, scientific, technical, administrative, and support service activities.
Source: Authors' calculation based on KLEMS data.

tivity effect. Germany, Italy and Japan also experienced a significant increase in the productivity effect and the price effect, but at the same time, they also sustained a large drop in the size effect. As a result, their accelerations in MFP growth were weaker than that for Canada. France also experienced a significant increase in the productivity effect, but the improvement was more than offset by a drop in the size effect, leading to a small deceleration in MFP growth. The United States had the largest deceleration in MFP growth mainly because it had the largest drop in the productivity effect.

The pattern of industry contributions to the change in MFP growth in Canada relative to other G7 countries over these two sub-periods (Table 2, Panel B) was generally similar to that for labour productivity growth. Again, it shows that the stabilization of manufacturing and the strength of FIRE allowed Canada to be able to overcome the largest drop in industry contribution from mining and quarrying and to enjoy the largest acceleration of MFP growth among G7 countries after 2010.

Table 3: Average Industry Share of Nominal GDP in Business Sector GDP in G7 Countries, 2000-2010 and 2011-2015 (per cent)

	Mining and Quarrying		Manufacturing		FIRE	
	2000-2010	2011-2015	2000-2010	2011-2015	2000-2012	2011-2015
Canada	9.5	9.9	19.0	14.2	15.2	15.5
France	0.1	0.1	14.6	12.4	17.7	18.8
Germany	0.2	0.2	23.9	24.3	17.3	16.5
Italy	0.5	0.4	19.0	17.0	18.4	21.1
Japan	0.1	0.1	27.4	26.4	11.2	10.8
UK	2.3	1.8	12.2	10.6	20.9	21.2
United States	2.0	2.9	15.3	14.2	21.8	22.0

Note: FIRE stands for financial, insurance and real estate activities; PSTAS stands for professional, scientific, technical, administrative, and support service activities.
Source: Authors' calculation based on KLEMS data.

Industry Perspectives: Mining and Quarrying, Manufacturing, and FIRE

In the world economy, economic activities are highly integrated across countries, especially those associated with mining and quarrying, manufacturing, and FIRE. Those industries also played a dominant role in the change in productivity growth before and after the global financial crisis. Compared to other G7 countries, mining and quarrying was the worst laggard to the revival of productivity growth in Canada while manufacturing was the largest positive contributor and FIRE was the second-largest positive contributor for Canada's higher productivity growth acceleration than other G7 countries except Germany.

The Canadian economy is much more dependant on resources than the other G7 countries. In terms of the share of nominal GDP in the business sector, mining and quarrying was much larger in Canada than in other G7 economies, and the importance increased slightly between the two sub-periods (Table 3).

Both manufacturing and FIRE each play important role in all economies, accounting for between 12 and 25 per cent of nominal business sector GDP. However, except in Germany and Japan, the importance of manufacturing declined in all the economies between 2000-2010 and 2011-2015. The decline was largest in Canada, from 19.0 per cent in 2000-2010 to 14.2 per cent in 2011-2015. The share of FIRE in those economies were fairly stable over the two sub-periods, and the largest positive change was in Italy from 18.4 per cent to 21.1 per cent.

The sharp decline in the manufacturing base in Canada is worrisome as the sector contributes significantly to employment, pays high wages and is the engine of the economy for innovation. Table 4 provides data on the shares of nominal business sector GDP for 10 manufacturing industries in Canada and in the United Sates in 2000,2010, and 2015. In total, the manufacturing share of nominal GDP declined by 10 percentage points between 2000 and 2015 in Canada, with all the decline taking place before 2010. The decline took place

12 For a detailed study of the productivity performance of the coke and refined petroleum industry, see Chan *et al.* (2014).

Table 4: **Manufacturing Industry Share of Business Sector Nominal GDP in Canada and the United States, 2000, 2010, and 2015 (%)**

	Canada			United States		
	2000	2010	2015	2000	2010	2015
Food, beverages and tobacco	2.66	2.49	2.26	1.85	1.81	1.66
Textiles, apparel and leather	0.95	0.24	0.18	0.56	0.21	0.18
Wood, paper and printing	4.06	1.76	1.61	1.51	0.92	0.83
Coke and refined petroleum	0.35	0.68	1.10	0.60	1.03	1.04
Chemicals	1.76	1.37	1.47	2.13	2.61	2.43
Rubber, plastics and non-metallic minerals	1.71	1.22	1.20	1.22	0.78	0.77
Basic and fabricated metal products	3.12	2.01	1.97	1.90	1.33	1.32
Electrical and optical equipment	2.22	1.01	0.74	3.06	2.36	2.16
Machinery and equipment	1.59	1.03	1.10	1.27	0.96	0.99
Transport equipment	5.08	2.03	2.37	2.36	1.62	1.90
Other manufacturing	1.08	0.77	0.67	1.05	0.82	0.72
Total Manufacturing	24.58	14.59	14.66	17.50	14.44	13.99

Source:Authors' calculation based on KLEMS data.

in all industries except coke and refined petroleum.[12] The industry with the largest decline was transport equipment (2.7 percentage points), followed by wood, paper and printing (2.5 percentage points) and electrical and optical equipment (1.5 percentage points). In contrast, the manufacturing share of GDP declined only 3.5 percentage points in total in the United States. Although the decline in the United States was also widespread across industries, the decline in each industry was relatively small, with the largest decline being 0.9 percentage points (electrical and optical equipment).

What were the driving forces of the decline of manufacturing in Canada in 2000-2010? Despite extensive research, the cause for the decline in manufacturing remains a puzzle. Capeluck (2015) provided some potential reasons. However, research suggests non-firm-specific factors might play a role as there was some evidence that the decline was mainly due to the weak performance of large and exporting manufacturing firms (Baldwin *et al.*, 2013; Tang 2014; Tang, 2017). Factors such as the negative impact of the sharp appreciation of Canadian dollar between 2002 and 2008, the weak demand from international markets (especially the U.S. market), and the increase in competition from emerging economies have been suggested to be the driving forces.

In next sub-sections of this section, we provide more evidence by comparing the economic performance of those three industries across G7 countries in terms of economic indicators that are important for productivity growth. We consider growth in real gross output, real gross output price, hours worked, labour productivity and MFP.

Mining and Quarrying

Compared to other industries, the developments in the mining and quarrying industry were highly volatile, mainly due to the fluctuation in commodity prices. The relative gross output price declined substantially in the second sub-period relative to in the first sub-period in all countries except in Japan after the global financial crisis (Table 5, Panel A). However, the change in other economic indicators were

Table 5: Output, Hours, Price and Productivity Trends in G7 Countries, 2000-2010 and 2011-2015 (Growth Rate in per cent per year)

Panel A: Mining and Quarrying

	Real Gross Output		Hours Worked		Real Gross Output Price		Labour Productivity		MFP	
	2000-2010	2011-2015	2000-2010	2011-2015	2000-2010	2011-2015	2000-2010	2011-2015	2000-2010	2011-2015
Canada	2.35	3.43	6.45	1.04	6.14	-4.12	-3.66	2.90	-3.64	-0.79
France	-2.37	-3.19	-2.73	-2.87	2.51	1.06	1.06	-0.24	-1.29	-1.10
Germany	-2.03	-4.33	-4.97	-4.48	1.69	-0.80	3.02	0.18	1.31	-0.86
Italy	-3.47	6.97	-1.92	-2.55	1.15	-2.17	-1.54	9.72	-2.63	4.70
Japan	-5.30	-2.00	-4.30	4.00	0.70	2.70	0.00	-4.40	-1.30	-2.00
UK	3.00	-6.10	-1.70	2.80	3.30	1.30	5.00	-8.10	3.60	-10.10
United States	1.20	5.33	2.37	3.76	8.71	-6.11	-0.79	1.92	0.88	3.42

Panel B: Manufacturing

	Real Gross Output		Hours Worked		Real Gross Output Price		Labour Productivity		MFP	
	2000-2010	2011-2015	2000-2010	2011-2015	2000-2010	2011-2015	2000-2010	2011-2015	2000-2010	2011-2015
Canada	-0.27	1.32	-2.18	-0.10	-1.11	0.99	1.87	1.42	0.02	0.51
France	0.04	0.55	-2.21	-0.95	-048	-0.08	2.27	1.51	0.48	0.42
Germany	1.94	1.98	-1.14	1.33	0.31	-0.75	3.03	0.62	0.74	0.38
Italy	0.31	-0.65	-1.56	-1.72	-0.15	-0.27	1.82	1.08	-0.06	0.12
Japan	-0.10	0.00	-2.10	-1.00	0.70	0.50	2.30	1.10	0.00	0.00
UK	-0.10	2.00	-4.20	0.40	1.20	0.80	4.30	1.70	1.00	0.50
United States	-0.38	2.40	-3.64	1.54	0.46	-0.96	3.37	0.85	1.00	-0.25

Panel C: FIRE

	Real Gross Output		Hours Worked		Real Gross Output Price		Labour Productivity		MFP	
	2000-2010	2011-2015	2000-2010	2011-2015	2000-2010	2011-2015	2000-2010	2011-2015	2000-2010	2011-2015
Canada	3.50	3.22	1.98	0.93	-0.39	0.62	0.56	1.70	0.06	1.14
France	2.40	1.17	0.62	0.39	0.19	-0.08	1.79	0.80	0.10	0.16
Germany	1.43	0.53	-0.85	-0.52	0.15	-0.88	2.30	1.07	-0.02	0.11
Italy	1.53	-0.01	0.72	-1.08	1.23	0.29	0.82	1.08	-0.31	0.28
Japan	0.50	1.60	-0.10	-1.30	-0.20	-1.50	0.60	3.10	-1.10	2.10
UK	3.60	2.20	1.00	0.70	1.30	0.60	2.50	1.50	0.50	1.20
United States	2.60	1.77	-0.08	1.43	-0.32	0.64	2.68	0.35	0.56	0.26

Source: Authors' calculation based on KLEMS data.

different across countries. Canada, Italy, and the United States increased real gross output growth and productivity growth in 2011-2015 compared to 2000-2010, but only Japan, the United Kingdom, and the United States managed a significant increase in employment growth. For France and Germany, growth in real gross output, employment or productivity slowed or was negative for most of the economic indicators.

Manufacturing

In general, manufacturing recovered in 2011-2015 from weakness in 2000-2010. The recovery in output and labour input was more dramatic in Canada, the United Kingdom and the United Sates and less so in France, Japan and Italy (Table 5, Panel B). Over these two sub-periods, all countries except Canada experienced a slowdown in both labour productivity and MFP growth. Canada also experienced a slowdown in labour productivity growth, but it managed to improve MFP in the second sub-period compared to a stagnant MFP performance in the previous sub-period.

FIRE

Compared to other countries, the strength of FIRE in Canada stood out across all indicators. Canada led G7 countries in hours worked growth in the 2000 to 2010 period. Despite slower growth after 2010, Canada maintained the highest growth rate in output in both 2000-2010 and 2011-2015 and second best growth rate in hours worked in 2011-2015 (Table 5, Panel C). In addition, Canada enjoyed faster labour productivity and MFP growth in 2011-2015 and had the second largest acceleration for both labour productivity growth and MFP growth, after Japan. The United States was the country with the largest slowdown in both labour productivity and MFP growth.

Conclusion

Canada has emerged as one of the better productivity growth performers among the G7 countries after the global financial crisis, which was in sharp contrast to its relatively weaker productivity growth performance in the 2000s. Canada's better productivity growth performance was due to Canada's improvement in productivity, but more importantly due to the slowdown in productivity growth in other G7 countries, especially in the United States.

Manufacturing played the most important role in Canada's superior productivity performance relative to other G7 countries after the global financial crisis, largely due to the much slower decline of the sector after 2010. Between these two sub-periods, the contribution of manufacturing to aggregate labour productivity growth increased 0.90 percentage points while in other countries, it ranged from -0.15 percentage points (Japan) to 0.45 percentage (France). Similarly, for MFP, the contribution in Canada increased 1.05 percentage points while in other countries, it ranged from 0.10 percentage points (Japan) to 0.50 percentage points (Italy). The evidence suggests that after a deep restructuring, the Canadian manufacturing sector has settled into a smaller, learner version of itself with an industry mix more aligned with its productive advantages.

Following manufacturing, Canada's relatively better performance in FIRE also contributed significantly to the Canada's productivity growth advantage over other countries. Over these two sub-periods, the contribution of FIRE to aggregate labour productivity growth increased 0.13 percentage points while in other countries, it ranged from -0.06 percentage points (Italy) to -0.49 percentage (the United States). Similarly, for MFP growth, the contribution of FIRE in Canada increased by 0.21 percentage points while in other countries, it ranged from 0.06 percentage points (Italy) to -0.25 percentage points (France). This reflects the strength of Canadian financial, insurance and real estate industries and the further improvement in productivity in this sector after the global financial crisis. This ultimately reflects Canada's sound banking system and better financial regulatory framework.

On the other hand, due to a sharp decline in commodity prices after the global financial crisis,[13] mining and quarrying made a much larger negative contribution to aggregate productivity growth after the global financial crisis in Canada than in the other G7 countries. Given Canada's reliance on the mining and quarry sector for economic growth, the volatility in the sector will continue to contribute to the change in Canada's productivity growth performance.

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13 See the Bank of Canada commodity price index (BCPI), which is an index of the spot or transaction prices in U.S. dollars of 26 commodities produced in Canada and sold in world markets (https://www.bankofcanada.ca/rates/price-indexes/bcpi/).

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Appendix: Methodology for the Decomposition of Aggregate Productivity Growth

In this Appendix, we develop the methodologies to decompose aggregate labour productivity growth and MFP growth into components at the industry level.

Decomposing Aggregate Labour Productivity Growth

Let V and V^r be nominal and real aggregate value-added, P^V be aggregate value added implicit price index, and L be aggregate labour input. The aggregate labour productivity (Ω) is defined as real aggregate value-added per unit of labour, i.e.

$$\Omega^V = V^r/L \tag{1}$$

Define V_i, Y_i^r, and P_i^Y as nominal value added, real gross output and the gross output price for industry i, respectively, and also M_i^r, P_i^M, and L_i as real intermediate inputs, intermediate input price and intermediate input labour, respectively.

With those definitions, the aggregate labour productivity can be traced back to components at the industry level as follows:

$$\begin{aligned}\Omega^V = \frac{V^r}{L} = \frac{V/P^V}{L} = \frac{\sum_i V_i}{P^V L} \\ = \frac{\sum_i (P_i^Y Y_i^r - P_i^M M_i^r)}{P^V L} \\ = \sum_i \left(\frac{P_i^Y}{P^V}\frac{Y_i^r}{L_i}\frac{L_i}{L} - \frac{P_i^M}{P^V}\frac{M_i^r}{L_i}\frac{L_i}{L}\right) \\ = \sum_i l_i (p_i^Y \Omega_i^Y - p_i^M \Omega_i^M)\end{aligned} \tag{2}$$

where $l_i = L_i/L$, $p_i^Y = P_i^Y/P^V$, $p_i^M = P_i^M/P^V$, $\Omega_i^Y = Y_i^r/L_i$, and $\Omega_i^M = M_i^r/L_i$. They represent the industry labour input share, the relative gross output price, the relative intermediate input price, gross output labour productivity and the intermediate input intensity of industry i, respectively.

Denote $\lambda_i^Y = l_i p_i^Y$ and $\lambda_i^M = l_i p_i^M$. Define the growth rate between years τ and t for any variable X as $\dot{X}_t = (T_t - T_\tau)/T_\tau$. Based on equation (2), aggregate labour productivity growth over a period (one year or more) from year τ to year t can be written as

$$\dot{\Omega}_t^V = \frac{\Omega_t^V - \Omega_\tau^V}{\Omega_\tau^V}$$

$$\begin{aligned}&= \frac{\sum_i [(\lambda_{it}^Y \Omega_{it}^Y - \lambda_{it}^M \Omega_{it}^M) - (\lambda_{i\tau}^Y \Omega_{i\tau}^Y - \lambda_{i\tau}^M \Omega_{i\tau}^M)]}{\Omega_\tau^V} \\ &= \sum_i [\lambda_{i\tau}^Y (\Omega_{it}^Y - \Omega_{i\tau}^Y) - \lambda_{i\tau}^M (\Omega_{it}^M - \Omega_{i\tau}^M) \\ &\quad + (\lambda_{it}^Y - \lambda_{i\tau}^Y)\Omega_{it}^Y - (\lambda_{it}^M - \lambda_{i\tau}^M)\Omega_{it}^M] \\ &= \sum_i [\lambda_{i\tau}^Y (\Omega_{i\tau}^Y/\Omega_\tau^V)\dot{\Omega}_{it}^Y - \lambda_{i\tau}^M (\Omega_{i\tau}^M/\Omega_\tau^V) \\ &\quad \dot{\Omega}_{it}^M] + \sum_i [(\Omega_{i\tau}^Y/\Omega_\tau^V)(1 + \dot{\Omega}_{it}^Y)\Delta\lambda_{it}^Y \\ &\quad - (\Omega_{it}^M/\Omega_\tau^V)(1 + \dot{\Omega}_{it}^M)\Delta\lambda_{it}^M]\end{aligned} \tag{3}$$

It can be derived that

$$\begin{aligned}w_{i\tau}^Y \cong \lambda_{i\tau}^Y (\Omega_{i\tau}^Y/\Omega_\tau^V) = \frac{L_{i\tau}}{L_\tau}\frac{P_{i\tau}^Y}{P_\tau^V}\frac{Y_{i\tau}^r/L_{i\tau}}{V_\tau^r/L_\tau} \\ w_{i\tau}^M \cong \lambda_{i\tau}^M (\Omega_{i\tau}^M/\Omega_\tau^V) = \frac{L_{i\tau}}{L_\tau}\frac{P_{i\tau}^M}{P_\tau^V}\frac{M_{i\tau}^r/L_{i\tau}}{V_\tau^r/L_\tau} \\ \sum_i (\Omega_{i\tau}^Y/\Omega_\tau^V)(1 + \dot{\Omega}_{it}^Y)\Delta\lambda_{it}^Y \\ = \sum_i w_{i\tau}^Y (1 + \dot{\Omega}_{it}^Y)(\dot{p}_{it}^Y + \dot{l}_{it} + \dot{p}_{it}^Y \dot{l}_{it}) \\ \sum_i (\Omega_{i\tau}^M/\Omega_\tau^V)(1 + \dot{\Omega}_{it}^M)\Delta\lambda_{it}^M \\ = \sum_i w_{i\tau}^M (1 + \dot{\Omega}_{it}^M)(\dot{p}_{it}^M + \dot{l}_{it} + \dot{p}_{it}^M \dot{l}_{it})\end{aligned} \tag{4}$$

So we have

$$\dot{\Omega}_t^V = \sum_i w_{i\tau}^Y[\dot{\Omega}_{it}^Y + (1+\dot{\Omega}_{it}^Y)(\dot{p}_{it}^Y + \dot{l}_{it} + \dot{p}_{it}^Y\dot{l}_{it})] - \sum_i w_{i\tau}^M[\dot{\Omega}_{it}^M + (1+\dot{\Omega}_{it}^M)(\dot{p}_{it}^M + \dot{l}_{it} + \dot{p}_{it}^M\dot{l}_{it})]$$
$$= \sum_i w_{i\tau}^Y[\dot{\Omega}_{it}^Y + \dot{p}_{it}^Y + \dot{l}_{it} + \dot{p}_{it}^Y\dot{l}_{it} + \dot{\Omega}_{it}^Y(\dot{p}_{it}^Y + \dot{l}_{it} + \dot{p}_{it}^Y\dot{l}_{it})] - \sum_i w_{i\tau}^M[\dot{\Omega}_{it}^M + \dot{p}_{it}^M + \dot{l}_{it} + \dot{p}_{it}^M\dot{l}_{it} + \dot{\Omega}_{it}^M(\dot{p}_{it}^M + \dot{l}_{it} + \dot{p}_{it}^M\dot{l}_{it})]$$
$$= \sum_i (w_{i\tau}^Y\dot{\Omega}_{it}^Y - w_{i\tau}^M\dot{\Omega}_{it}^M) + \sum_i (w_{i\tau}^Y\dot{p}_{it}^Y - w_{i\tau}^M\dot{p}_{it}^M) + \sum_i (w_{i\tau}^Y - w_{i\tau}^M)\dot{l}_{it} + \sum_i \{w_{i\tau}^Y[\dot{p}_{it}^Y\dot{l}_{it} + \dot{\Omega}_{it}^Y(\dot{p}_{it}^Y + \dot{l}_{it} + \dot{p}_{it}^Y\dot{l}_{it})] - w_{i\tau}^M[\dot{p}_{it}^M\dot{l}_{it} + \dot{\Omega}_{it}^M(\dot{p}_{it}^M + \dot{l}_{it} + \dot{p}_{it}^M\dot{l}_{it})]\} \quad (5)$$

Decomposing Aggregate MFP Growth

Let I be aggregate input.[14] Aggregate MFP is defined as real value added per unit of aggregate input:

$$X = V^r/I \quad (6)$$

Denote P^I as the aggregate input price and and as total input and its price for industry i. With these definitions, the aggregate MFP can be traced back to components at the industry level as follows:

$$X = \frac{V^r}{I} = \frac{V}{P^V I} = \frac{\sum_i (Y_i - M_i)}{P^V I} = \frac{\sum_i (P_i^Y y_i^r - P_i^M M_i^r)}{P^V I}$$
$$= \sum_i \left(\frac{P_i^Y/P_i^I}{P^V/P^I}\frac{Y_i^r}{I_i}\frac{P_i^I I_i}{P^I I} - \frac{P_i^M/P_i^I}{P^V/P^I}\frac{M_i^r}{I_i}\frac{P_i^I I_i}{P^I I}\right)$$
$$= \sum_i \Theta_i(\omega_i^Y X_i^Y - \omega_i^M X_i^M) \quad (7)$$

where $X_i^Y \cong Y_i^r/I_i$, $X_i^M \cong M_i^r/I_i$, $\Theta_i \cong \frac{P_i^I I_i}{P^I I}$, $\omega_i^Y \cong \frac{P_i^Y/P_i^I}{P^V/P^I}$, and $\omega_i^M \cong \frac{P_i^M/P_i^I}{P^V/P^I}$.

They represents real output per unit of total input (or MFP), intermediate input intensity (in all inputs), the ratio of industry total input cost to aggregate total input cost, the ratio of industry relative output price (P_i^Y/P^V) to the industry real total input price (P_i^I/P^I), and the ratio of the industry real intermediate input price (P_i^M/P^V) to the industry real total input price.[15]

14 The aggregate input index can be measured as a Törnqvist index, $\Delta\ln(I_t) = \sum_j \bar{w}_{jt}\Delta\ln(I_{jt})$, of labour, capital and intermediate inputs, with $\bar{w}_{jt}$ being the two-period average cost share of the corresponding input. For the aggregate input, the aggregate input index is then indexed to the total input costs in the base year.

15 We call P_i^Y/P^V industry real input price, P_i^I/P^I industry input price, and P_i^M/P^V industry real intermediate price as the industry prices are deflated by aggregate prices.

Define $s_i^Y = \Theta_i \bar{\omega}_i^Y$ and $s_i^M = \Theta_i \bar{\omega}_i^M$. The aggregate MFP growth over a period from year τ to year t (one year or more) can be written as

$$
\begin{aligned}
X_t^{\&} &= \frac{X_t - T_\tau}{X_\tau} \\
&= \frac{\sum_i [(s_{it}^Y X_{it}^Y - s_{it}^M X_{it}^M) - (s_{i\tau}^Y X_{i\tau}^Y - s_{i\tau}^M X_{i\tau}^M)]}{X_\tau} \\
&= \frac{\sum_i [s_{i\tau}^Y (X_{it}^Y - X_{i\tau}^Y) - s_{i\tau}^M (X_{it}^M - X_{i\tau}^M) + (s_{it}^Y - s_{i\tau}^Y) X_{it}^Y - ((s_{it}^M - s_{i\tau}^M) X_{it}^M]}{X_\tau} \\
&= \sum_i [s_{i\tau}^Y (X_{i\tau}^Y / X_\tau) \dot{X}_{it}^Y - s_{i\tau}^M (X_{i\tau}^M / X_\tau) \dot{X}_{it}^M] \\
&\quad + \sum_i [(X_{i\tau}^Y / X_\tau)(1 + \dot{X}_{it}^Y) \Delta s_{i\tau}^Y \\
&\quad - (X_{i\tau}^M / X_\tau)(1 + \dot{X}_{it}^M) \Delta s_{i\tau}^M] \quad (8)
\end{aligned}
$$

It can be derived that

$$
\begin{aligned}
w_{i\tau}^Y &\cong s_{i\tau}^Y (X_{i\tau}^Y / X_\tau) = \frac{P_{i\tau}^Y Y_{i\tau}^r}{P_\tau^V V_\tau^r}, \\
w_{i\tau}^M &\cong s_{i\tau}^M (X_{i\tau}^M / X_\tau) = \frac{P_{i\tau}^M M_{i\tau}^r}{P_\tau^V V_\tau^r}, \\
&\sum_i (X_{i\tau}^Y / X_\tau)(1 + \dot{X}_{it}^Y) \Delta s_{it}^Y \\
&= \sum_i w_{i\tau}^Y (1 + \dot{X}_{it}^Y)(\dot{\omega}_{it}^Y + \dot{\Theta}_{it} + \dot{\omega}_{it}^Y \dot{\Theta}_{it}), \\
&\sum_i (X_{i\tau}^M / X_\tau)(1 + \dot{X}_{it}^M) \Delta s_{it}^M \\
&= \sum_i w_{i\tau}^M (1 + \dot{X}_{it}^M)(\dot{\omega}_{it}^M + \dot{\Theta}_{it} + \dot{\omega}_{it}^M \dot{\Theta}_{it}) \quad (9)
\end{aligned}
$$

Thus, equation (8) becomes

$$
\begin{aligned}
\dot{X}_t &= \sum_i w_{i\tau}^Y [\dot{X}_{it}^Y + (1 + \dot{X}_{it}^Y) \\
&\quad (\dot{\omega}_{it}^Y + \dot{\Theta}_{it} + \dot{\omega}_{it}^Y \dot{\Theta}_{it})] \\
&\quad - \sum_i w_{i\tau}^M [\dot{X}_{it}^M + (1 + \dot{X}_{it}^M) \\
&\quad (\dot{\omega}_{it}^M + \dot{\Theta}_{it} + \dot{\omega}_{it}^M \dot{\Theta}_{it})] \\
&= \sum_i w_{i\tau}^Y [\dot{X}_{it}^Y + \dot{\omega}_{it}^Y + \dot{\Theta}_{it} + \dot{\omega}_{it}^Y \dot{\Theta}_{it} \\
&\quad + \dot{X}_{it}^Y (\dot{\omega}_{it}^Y + \dot{\Theta}_{it} + \dot{\omega}_{it}^Y \dot{\Theta}_{it})] \\
&\quad - \sum_i w_{i\tau}^M [\dot{X}_{it}^M + \dot{\omega}_{it}^M + \dot{\Theta}_{it} + \dot{\omega}_{it}^M \dot{\Theta}_{it} \\
&\quad + \dot{X}_{it}^M (\dot{\omega}_{it}^M + \dot{\Theta}_{it} + \dot{\omega}_{it}^M \dot{\Theta}_{it})] \\
&= \sum_i (w_{i\tau}^Y \dot{X}_{it}^Y - w_{i\tau}^M \dot{X}_{it}^M) \\
&\quad + (w_{i\tau}^Y \dot{\omega}_{it}^Y - w_{i\tau}^M \dot{\omega}_{it}^M) \\
&\quad + \sum_i (w_{i\tau}^Y - w_{i\tau}^M) \dot{\Theta}_{it} \\
&\quad + \sum_i \{ w_{i\tau}^Y [\dot{\omega}_{it}^Y \dot{\Theta}_{it} + \dot{X}_{it}^Y (\dot{\omega}_{it}^Y + \dot{\Theta}_{it} + \\
&\quad \dot{\omega}_{it}^Y \dot{\Theta}_{it})] - w_{i\tau}^M [\dot{\omega}_{it}^M \dot{\Theta}_{it} \\
&\quad + \dot{X}_{it}^M (\dot{\omega}_{it}^M + \dot{\Theta}_{it} + \dot{\omega}_{it}^M \dot{\Theta}_{it})] \} \quad (10)
\end{aligned}
$$