A Appendix

Our starting point is:

$$\text{Productivity} = \frac{r_{GDP}GDP + r_{WBO}WBO + r_{ANS}ANS}{w_K K + w_L L + w_{WBI}WBI}. \tag{11}$$

The problem with equation (11) is the computation of weights (r_{GDP}, r_{WBO}, r_{ANS}, w_K, w_L, w_{WBI}). Data Envelopment Analysis is a convenient framework to find optimal values for the weights without additional information (in particular if prices are not available). The idea is to compute optimal weights so that the ratio of equation (11), for a country labelled 0, is as large as possible, and the same ratios for all other countries are positive and below 1. As a consequence, countries with the highest ratios will have an optimal value of 1, and the closest to zero the ratio is, the lowest the efficiency is. This method allows to benchmark countries with respect to the most efficient ones. Formally, we have the following fractional program:

$$\max_{(r_{GDP}, r_{WBO}, r_{ANS}, w_K, w_L, w_{WBI})} \frac{r_{GDP}GDP_0 + r_{WBO}WBO_0 + r_{ANS}ANS_0}{w_K K_0 + w_L L_0 + w_{WBI}WBI_0}$$

$$s.t. \frac{r_{GDP}GDP_j + r_{WBO}WBO_j + r_{ANS}ANS_j}{w_K K_j + w_L L_j + w_{WBI}WBI_j} \leq 1, j = 1, ..., N$$

$$r_{GDP}, r_{WBO}, r_{ANS}, w_K, w_L, w_{WBI} \geq 0$$

This model can be converted into a linear program model, as follows: let $t = 1/(w_K K_j + w_L L_j + w_{WBI}WBI_j)$, then the previous fractional program becomes:

$$\max_{(r_{GDP},r_{WBO},r_{ANS},w_K,w_L,w_{WBI})} t(r_{GDP}GDP_0 + r_{WBO}WBO_0 + r_{ANS}ANS_0)$$
$$t(w_K K_0 + w_L L_0 + w_{WBI}WBI_0) = 1$$
$$s.t.$$
$$t(r_{GDP}GDP_j + r_{WBO}WBO_j + r_{ANS}ANS_j)$$
$$- t(w_K K_j + w_L L_j + w_{WBI}WBI_j) \leq 0, j = 1, ..., N$$
$$r_{GDP}, r_{WBO}, r_{ANS}, w_K, w_L, w_{WBI} \geq 0$$
$$t \geq 0$$

Changing notation, $tr_y = u_y$ and $tw_x = v_x$ ($y \in \{GDP, WBO, ANS\}, x \in \{K, L, WBI\}$), then:

$$\max_{(u_{GDP},u_{WBO},u_{ANS},v_K,v_L,v_{WBI})} u_{GDP}GDP_0 + u_{WBO}WBO_0 + u_{ANS}ANS_0$$
$$v_K K_0 + v_L L_0 + v_{WBI}WBI_0 = 1$$
$$s.t.$$
$$(u_{GDP}GDP_j + u_{WBO}WBO_j + u_{ANS}ANS_j)$$
$$- (v_K K_j + v_L L_j + v_{WBI}WBI_j) \leq 0, j = 1, ..., N$$
$$u_{GDP}, u_{WBO}, u_{ANS}, v_K, v_L, v_{WBI} \geq 0$$

Last, this linear program has a dual representation:

$$
\begin{aligned}
&\min_{\lambda_j} \theta_0 \\
&\sum_j \lambda_j K_j \leq \theta_0 K_0 \\
&\sum_j \lambda_j L_j \leq \theta_0 L_0 \\
&\sum_j \lambda_j WBI_j \leq \theta_0 WBI_0 \\
&\sum_j \lambda_j GDP_j \geq GDP_0 \\
&\sum_j \lambda_j WBO_j \geq WBO_0 \\
&\sum_j \lambda_j ANS_j \geq ANS_0 \\
&\lambda_j \geq 0
\end{aligned}
$$

In this model any improvement in productivity can only be obtained by decreasing the use of inputs. In this case, we speak of an input oriented model. Alternatively, one may be interested in assessing to what extent outputs can be increased given the use of inputs. In this case, we refer to the output oriented model:

$$
\begin{aligned}
&\max_{\lambda_j} \phi_0 \\
&\sum_j \lambda_j K_j \leq K_0 \\
&\sum_j \lambda_j L_j \leq L_0 \\
&\sum_j \lambda_j WBI_j \leq WBI_0 \\
&\sum_j \lambda_j GDP_j \geq \phi_0 GDP_0 \\
&\sum_j \lambda_j WBO_j \geq \phi_0 WBO_0 \\
&\sum_j \lambda_j ANS_j \geq \phi_0 ANS_0 \\
&\lambda_j \geq 0
\end{aligned}
$$

This model is the starting point of the procedure developed by Toloo et al. (2021) to select variables.