Labour Productivity as a Measure of Technological Change

Ulrich Kohli[1]
University of Geneva

Abstract

Average labour productivity (ALP) is today the productivity measure most used by policy makers, the media, and the general public. Economists recognize however that it is an inadequate measure of technological change. This is because ALP is a hybrid measure that captures both shifts in the production possibilities frontier and movements along the frontier itself. Thus, the flaw of ALP as a measure of technological change is not that it uses labour as a benchmark, which is a perfectly appropriate, but that, by being a partial measure of productivity, it ignores the role of capital, not just when accounting for technological change, but, even much more seriously, in production altogether. Put in other words, the numerator of the ALP ratio is not consistent with its denominator as a measure of technological change, and it is not the denominator that is at fault, but the numerator. A complete, or total measure of labour productivity (TLP) is therefore proposed and compared to the ALP and the better-known total factor productivity (TFP) measures. The relationship between the three productivity measures can also be analyzed in the dual price space. Numerical results for the U.S. private nonfarm business sector are provided as an illustration.

Output or gross domestic product (GDP) per unit of labour is today the aggregate measure of productivity most used by policy makers, the media, and the general public. Yet, average labour productivity (ALP) so defined has somewhat of a bad press among economists.[2] For a start, by crediting the totality of productivity gains to labour and thus overlooking the contribution of capital, it seems rather one-sided. Much more importantly, though, ALP is fundamentally a partial measure of productivity, to use the terminology of Domar (1962), and it is therefore not fit to measure technological change. This is why economists tend to favour the concept of

1 Emeritus Professor of Economics, Geneva School of Economics and Management, University of Geneva, Geneva, Switzerland. I am grateful to W. Erwin Diewert for many helpful comments and suggestions on my earlier work on this and related topics, and to Paul Schreyer and Andrew Sharpe as well as two anonymous referees for their insights and advice on an earlier draft of this paper. Any errors or omissions are mine alone. Email: Ulrich.Kohli@hotmail.com, Ulrich.Kohli@unige.ch.

2 The wide acceptance of this concept probably has to do in part to its early adoption by the Organisation for European Economic Co-operation (OEEC, the ancestor of the Organisation for Economic Co-operation and Development, OECD) in 1949 under the influence of Jean Fourastié; see Boulat (2006: 97).

total factor productivity (TFP) that takes into account of both labour and capital.[3]

ALP is somewhat of a hybrid measure of productivity growth. It is well known that it combines elements of technological progress – which in the context of a simple two-input, one-output production function can be thought of as an upward shift in the production function – and effects of technical change – which can be described as changes in the input mix, i.e. movements along an isoquant.[4]

Economists are mostly interested in the progression of productivity over time. What matters then, when comparing different measures, is not the size of the numerators and denominators, but the growth rates of their components. With capital services typically increasing more rapidly than employment, and indeed output, ALP growth will tend to exceed TFP growth, whereas *average capital productivity* (AKP) growth will not just fall short of TFP growth, but it will even tend to be negative. This does not make AKP a very appealing concept; hence it is its inverse – which is a measure of the capital intensity of production (KIP) and usually is increasing over time – that receives more attention.

Even though ALP is not an appropriate measure of technological progress, the focus on labour should not be rejected out of hand. This would be tantamount to throwing out the baby with the bathwater, to use a metaphor dear to economists. There might be good reasons to focus on labour. For one thing, labour is more intuitive a concept and better understood by the wide public than physical capital. Even though output per unit of labour is not the same as income per capita, labour productivity is often used as a welfare indicator or a measure of economic development.[5]

Furthermore, labour can be viewed as the only true primary factor of production – since capital has been produced by labour in previous periods – and thus it is the ultimate force behind production, technological change, and growth.

Technological change is often modelled as being disembodied and factor augmenting. It is as if an available endowment of labour and capital increased with the passage of time when measured in terms of efficiency units. If labour and capital both benefit from efficiency gains at the same rate, technological change is said to be balanced, or Hicks-neutral. If the passage of time benefits labour exclusively, technological change is said to be labour-augmenting, or Harrod-neutral, and if capital is the sole beneficiary, techno-

3 The term total factor productivity seems to have been introduced by Kendrick (1961); see Domar (1962) who prefers the name The Residual; this term is also known as multifactor productivity (MFP), and it is often referred to as the Solow (1957) residual when derived from econometric estimates.

4 Technological progress is sometimes described as increases in knowledge, whereas technical change may be thought of as changes in processes. Technical change is often incremental and internal to firms, whereas technological change is a broader process and it may imply a shift in the technological paradigm. While this distinction is fuzzy, it is convenient to distinguish the two types of changes that can occur in the context of our model. See Nelson and Winter (1982), Rosenberg (1983), and Freeman and Soete (1997) for in-depth analyses.

5 A high average labour productivity in international comparison does not necessarily mean that the country's workers are better skilled or more hard working: it might simply mean that the have more capital to work with, and this capital goes unaccounted for.

logical change is described as being capital–augmenting, or Solow-neutral. There is some evidence that technological change tends to be largely labour augmenting (i.e. coming close to being Harrod-neutral), thus yet another argument in favour of the use of labour as a yardstick.[6]

TFP, on the other hand, models technological change as if it were Hicks-neutral. In any case, one should be able to use any benchmark one pleases. Technological change can be measured from different perspectives, but it is important to be fully aware of the standpoint one takes. We will argue that labour is a perfectly appropriate reference, but that ALP is generally not an acceptable measure of technological change, not because it neglects the role of capital in that process, but, much more importantly, because it actually ignores the contribution of capital to current production altogether. The numerator and denominators of ALP are not consistent with one another for the purpose of measuring technological change, and, moreover, it is not the denominator that is at fault (if it is selected by design), but the numerator. In lieu of a partial productivity measure such as ALP, what is needed is a complete, i.e. total productivity measure, one that takes all inputs and outputs into account, even though the focus is on labour.

In order to present our argument in the simplest possible way, we will use here a very basic representation of the technology that could be thought of as being modelled by a one-output, two-input (labour and capital) aggregate production function. Our approach, however, can easily be generalized to include more inputs and/or outputs.[7]

As usual when modelling technologies at the aggregate or national level, we will assume constant returns to scale, perfect information and foresight, optimization, the absence of adjustment costs and full capacity utilization, perfect competition, the absence of measurement errors as well as of externalities such as environmental issues, but these assumptions could be relaxed if needed at the firm or industry level. We will apply measurement theory to obtain a total labour productivity (TLP) – or Harrodneutral – index of technological change and we will show how it differs from the well-known TFP index. We will further show that TLP, by netting out the contribution of capital, is better related to the evolution of real wages than is ALP. This is all the more relevant since the growth in ALP is often used as a benchmark by policy makers and employers alike when assessing the inflationary potential of nominal wage increases. TLP can be thought of as an upper bound of the growth of real wages made possible by technological progress, abstracting from the effect of changes in factor intensities. Actual measures for the United States will be reported as an illustration.

The article contains five sections and proceeds as follows. In the first section, we

6 See Kohli (1991, 2010, 2015) for some evidence for the United States, for instance, and Chambers (1988) for a good theoretical review of the different forms of technological change.

7 See Kohli (2025) for a more general model with the focus on imports.

review the relationship between ALP and TFP. In the following section, we formally define TLP, and we show how it relates to ALP and TFP. In the third section we re-examine the relationship between all three productivity measures in the dual, price space; this will also enable us to bring the marginal labour and capital productivity concepts into the analysis. This is followed by an empirical illustration, using data for the U.S. private nonfarm business sector. The last section concludes.

Average Labour and Total Factor Productivity

Consider a simple one-output, two-input technology. The quantity of output at time t, which could be thought of as real GDP, is denoted by $q_{Y,t}$, and its price by $p_{Y,t}$. We assume two factors of production, capital (K) and labour (L); the corresponding quantities at time t are denoted $x_{K,t}$ and $x_{L,t}$, with rental prices $w_{K,t}$ and $w_{L,t}$.[8]

We begin with the national accounts identity that implies the equality between the country's nominal output (Y_t) and factor payments:[9]

$$Y_t = p_{Y,t} q_{Y,t} = w_{K,t} x_{K,t} + w_{L,t} x_{L,t} \tag{1}$$

Let $Q_{Y,t,t-1}$ and $P_{Y,t,t-1}$ denote the growth factors (one plus the rate of growth between time $t-1$ and time t) of real output and its price:

$$Q_{Y,t,t-1} = \frac{q_{Y,t}}{q_{Y,t-1}}, \quad P_{Y,t,t-1} = \frac{p_{Y,t}}{p_{Y,t-1}} \tag{2}$$

Similarly, we define the growth factors of the quantities and prices of capital and labour:

$$\begin{aligned} X_{K,t,t-1} &= \frac{x_{K,t}}{x_{K,t-1}}, & X_{L,t,t-1} &= \frac{x_{L,t}}{x_{L,t-1}}, \\ W_{K,t,t-1} &= \frac{w_{K,t}}{w_{K,t-1}}, & W_{L,t,t-1} &= \frac{w_{L,t}}{w_{L,t-1}} \end{aligned} \tag{3}$$

The ALP, AKP and KIP indices can now be obtained as:

$$\begin{aligned} ALP_{t,t-1} &= \frac{Q_{Y,t,t-1}}{X_{L,t,t-1}}, \quad AKP_{t,t-1} = \frac{Q_{Y,t,t-1}}{X_{K,t,t-1}}, \\ KIP_{t,t-1} &= \frac{X_{K,t,t-1}}{Q_{Y,t,t-1}} \end{aligned} \tag{4}$$

Let $s_{K,t}$ and $s_{L,t}$ be the cost shares of capital and labour in production at time t:

$$s_{K,t} = \frac{w_{K,t} x_{K,t}}{p_{Y,t} q_{Y,t}}, \quad s_{L,t} = \frac{w_{L,t} x_{L,t}}{p_{Y,t} q_{Y,t}} \tag{5}$$

with $s_{K,t} + s_{L,t} = 1$, and define $\bar{s}_{K,t,t-1}$ and $\bar{s}_{L,t,t-1}$ as their averages over consecutive periods:

$$\bar{s}_{K,t,t-1} = \frac{s_{K,t} + s_{K,t-1}}{2}, \bar{s}_{L,t,t-1} = \frac{s_{L,t} + s_{L,t-1}}{2} \tag{6}$$

We next define $X_{t,t-1}$ as a Törnqvist in-

8 The return to capital is measured on an ex-post basis as it is typically done when dealing with country-wide data and while assuming perfect foresight and the absence of adjustment costs; relaxing these assumptions would undeniably impact on the actual measurement of TLP, just like it has been shown to affect measures of TFP; see Berndt and Fuss (1986), Hulten (1986), and Oulton (2007).

9 Indirect taxes and subsidies are netted out of output prices to ensure this equality.

put quantity index:[10]

$$X_{t,t-1} = X_{K,t,t-1}^{\bar{s}_{K,t,t-1}} \cdot X_{L,t,t-1}^{\bar{s}_{L,t,t-1}} \qquad (7)$$

We then obtain the TFP index as:

$$TFP_{t,t-1} = \frac{Q_{Y,t,t-1}}{X_{t,t-1}} \qquad (8)$$

The relationship between $ALP_{t,t-1}$ and $TFP_{t,t-1}$ can easily be derived:

$$\begin{aligned} ALP_{t,t-1} &= \frac{Q_{Y,t,t-1}}{X_{L,t,t-1}} \\ &= \frac{Q_{Y,t,t-1}}{X_{K,t,t-1}^{\bar{s}_{K,t,t-1}} X_{L,t,t-1}^{\bar{s}_{L,t,t-1}}} \cdot \frac{X_{K,t,t-1}^{\bar{s}_{K,t,t-1}} X_{L,t,t-1}^{\bar{s}_{L,t,t-1}}}{X_{L,t,t-1}} \\ &= \frac{Q_{Y,t,t-1}}{X_{t,t-1}} \cdot \left(\frac{X_{K,t,t-1}}{X_{L,t,t-1}}\right)^{\bar{s}_{K,t,t-1}} \\ &= TFP_{t,t-1} \cdot \left(\frac{X_{K,t,t-1}}{X_{L,t,t-1}}\right)^{\bar{s}_{K,t,t-1}} \end{aligned} \qquad (9)$$

This result is well known: ALP growth is the resultant of TFP growth (technological change) and of the impact of increases in the capital-labour intensity ratio (technical change). Since the capital-labour ratio tends to increase over time, ALP growth will typically exceed TFP growth.

Total Labour Productivity

There is an extensive literature about measuring productivity in the presence of intermediate goods or services.[11] Most of this literature deals with sectorial or industrial analyses, where intermediate inputs might be energy, materials, and purchased services. Aggregation over sectors and industries must be carried out with special care in order to avoid any double accounting. The issue of double accounting does normally not arise at the national level since domestic intermediate goods then typically wash out.[12] Nonetheless, a complete accounting of production and productivity gains at the national level requires that all primary inputs and outputs be taken into account.

In our case of interest, the issue is not so much one of double accounting, but rather one of accounting omission. Many national statistical agencies publish ALP and TFP statistics, often at the aggregate and at the industry level. An important issue at the industry level is how is "output" measured? Is it a gross output measure, that takes all inputs into consideration, or is it a value-added measure that nets out intermediate inputs, i.e. treats them as negative outputs?[13]

In the first case, TFP will be measured relative to an aggregate or weighted growth rate of labour, capital, and intermediate goods, whereas in the latter case the de-

10 The Törnqvist index is a superlative index and it has been shown by Diewert (1974, 1976) to be exact for the Translog functional form introduced by Christensen, Jorgenson, and Lau (1973). It is typically numerically very close to another superlative index, the Fisher almost ideal index; see Diewert (1978).

11 See Kendrick (1961), Domar (1961, 1962), Binswanger (1974), Berndt and Wood (1975, 1982), Hulten (1978), Gollop (1979), Jorgenson and Fraumeni (1981), Jorgenson, Gollop, and Fraumeni (1987), Balk (2009, 2010), for instance.

12 Imported intermediate inputs are subtracted from gross output when calculating GDP.

13 The OECD offers some valuable guidelines in this respect; see OECD (2001, 2023).

nominator of TFP will be an aggregate of just labour and capital. Each approach has some advantages and some drawbacks,[14] but they both make perfect sense. It is therefore all the more surprising that the same logic is not being followed when it comes to measuring labour productivity.[15] Independently of whether intermediate inputs are included or left out, the numerator – gross output, or value added by labour and capital, as it may be – is not consistent in an accounting sense with the denominator (labour). The same is true at the aggregate level, where intermediate inputs are not an issue, but where the numerator is typically a gross output measure, whereas it should be a value-added measure after having netted out capital services.

In order to obtain an appropriate labour measure of technological change one needs a *total* measure that is compatible with accounting identity (1) above. That is, the contribution of capital to production cannot just be ignored: it must be netted out. In other words, one must treat capital services as an intermediate input, i.e. a negative output. This is not to say that we generally view labour as a fixed input and capital as a variable one, which would make little sense from a national or a firm viewpoint in the short run, but simply that the numerator should be consistent with the primary input variable that is being used as the denominator in calculating productivity.

Let us begin by rewriting accounting identity (1) as follows, thereby defining Λ_t as nominal *net output*, i.e. nominal value added by labour (equivalently, the wage bill):

$$\Lambda_t = p_{Y,t}q_{Y,t} - w_{K,t}x_{K,t} = w_{L,t}x_{L,t} \quad (10)$$

Treating capital as a negative output can be thought of as replacing the production function implicit throughout Section 2 by a real value-added function, a special case of a variable profit function (Diewert 1974, 1978). We then need a measure of net output, i.e. real value added by labour. The value shares of gross output and capital in net output are:

$$\lambda_{Y,t} = \frac{p_{Y,t}q_{Y,t}}{\Lambda_t} > 1, \quad \lambda_{K,t} = \frac{w_{K,t}x_{K,t}}{\Lambda_t} > 0 \quad (11)$$

with $\lambda_{Y,t} - \lambda_{K,t} = 1$. The Törnqvist quantity index of *net output*, i.e. real value added by labour, then is:

$$Q_{\Lambda,t,t-1} = Q_{Y,t,t-1}^{\bar{\lambda}_{Y,t,t-1}} \cdot X_{K,t,t-1}^{-\bar{\lambda}_{K,t,t-1}} \quad (12)$$

where $\bar{\lambda}_{Y,t,t-1}$ and $\bar{\lambda}_{K,t,t-1}$ are the average value shares over consecutive periods.[16]

We then can define the *total labour pro-*

14 See Schreyer (2001) for a good discussion.

15 The same is typically true when it comes to measuring capital productivity, although we are aware of two exceptions. Thus, Lawrence, Diewert, and Fox (2006) compute what they call a capital TFP measure. Similarly, Balk (2010) derives a number of capital productivity measures, treating labour as a negative output; nonetheless, he does not follow the same approach when defining labour productivity.

16 Quantity index (12) would be exact for the representation of the technology by a translog variable profit function treating capital as a negative output; see Diewert (1974, 1982, 2022).

ductivity (TLP) growth factor as:[17]

$$TLP_{t,t-1} = \frac{Q_{\Lambda,t,t-1}}{X_{L,t,t-1}} \qquad (13)$$

The difference between ALP and TLP is thus that in the latter case the real contribution of capital to production is netted out. This is as if, in the Laspeyres case, the constant dollar value of capital services were being subtracted from real gross output. The quantity of capital services generally increases more rapidly than real gross output, so that the growth in real net output will be that much reduced. Consequently, one should typically expect TLP to grow at a slower rate than ALP.

The question now obviously arises as to the relationship between the TLP and TFP measures. Note that it follows from (5) and (11) in view of (1) that:

$$\lambda_{Y,t} = \frac{1}{1-s_{K,t}} = \frac{1}{s_{L,t}}, \quad \lambda_{K,t} = \frac{s_{K,t}}{1-s_{K,t}} = \frac{s_{K,t}}{s_{L,t}} \qquad (14)$$

In terms of averages over consecutive periods we use the following approximations:

$$\bar{\lambda}_{Y,t,t-1} = \frac{1}{2}\left(\frac{1}{s_{L,t}} + \frac{1}{s_{L,t-1}}\right) = \frac{\bar{s}_{L,t,t-1}}{s_{L,t}s_{L,t-1}} \simeq \frac{1}{\bar{s}_{L,t,t-1}} \qquad (15)$$

$$\bar{\lambda}_{K,t,t-1} = \bar{\lambda}_{Y,t,t-1} - 1 \simeq \frac{1}{\bar{s}_{L,t,t-1}} - 1 = \frac{\bar{s}_{K,t,t-1}}{\bar{s}_{L,t,t-1}} \qquad (16)$$

We thus find:[18]

$$\begin{aligned} TLP_{t,t-1} &= Q_{Y,t,t-1}^{\bar{\lambda}_{Y,t,t-1}} X_{K,t,t-1}^{-\bar{\lambda}_{K,t,t-1}} X_{L,t,t-1}^{-1} \\ &= \left[Q_{Y,t,t-1} \cdot X_{K,t,t-1}^{-\frac{\bar{\lambda}_{K,t,t-1}}{\bar{\lambda}_{Y,t,t-1}}} \cdot X_{L,t,t-1}^{-\frac{1}{\bar{\lambda}_{Y,t,t-1}}} \right]^{\bar{\lambda}_{Y,t,t-1}} \\ &\simeq \left(Q_{Y,t,t-1} X_{K,t,t-1}^{-\bar{s}_{K,t,t-1}} X_{L,t,t-1}^{-\bar{s}_{L,t,t-1}} \right)^{\bar{\lambda}_{Y,t,t-1}} \\ &= TFP_{t,t-1}^{\bar{\lambda}_{Y,t,t-1}} \end{aligned} \qquad (17)$$

Since $\bar{\lambda}_{Y,t,t-1} > 1$, TLP will tend to exceed TFP whenever technological change is positive. This is not surprising, since technological progress is fully allocated to labour by design. It is important to stress that, unlike ALP, TLP is a complete measure of technological change in that it takes the contribution of capital to production into account. Like TFP, TLP does not depend on the change in the capital-labour ratio over time. Both TFP and TLP measure the shift in the production possibilities frontier, independently of the capital-labour ratio, albeit from different perspectives.[19]

We may now examine the relationship between TLP and ALP. From (9) and (17)

17 As suggested earlier, the adjective *total* is used to indicate that TLP is a *complete* or *comprehensive* measure of technological change, one that takes *all* inputs and outputs into account; it could also be described as a *Harrod* measure since it imputes all technological change to labour.

18 We have verified in our empirical illustration of Section 5 below that this approximation holds to at least the fourth decimal point.

19 See Kohli (2025) for further perspectives in an open economy context.

we find:

$$
\begin{aligned}
ALP_{t,t-1} &= \frac{Q_{Y,t,t-1}}{X_{L,t,t-1}} \\
&= \frac{Q_{Y,t,t-1}^{1+\bar{\lambda}_{K,t,t-1}} \cdot Q_{Y,t,t-1}^{-\bar{\lambda}_{K,t,t-1}}}{X_{L,t,t-1}} \\
&\quad \cdot X_{K,t,t-1}^{-\bar{\lambda}_{K,t,t-1}} \cdot X_{K,t,t-1}^{\bar{\lambda}_{K,t,t-1}} \\
&= \frac{Q_{\Lambda,t,t-1}}{X_{L,t,t-1}} \cdot \left(\frac{X_{K,t,t-1}}{Q_{Y,t,t-1}} \right)^{\bar{\lambda}_{K,t,t-1}} \\
&= TLP_{t,t-1} \cdot KIP_{t,t-1}^{\bar{\lambda}_{K,t,t-1}}
\end{aligned} \tag{18}
$$

where KIP is again the capital intensity of production index as defined in (4). Thus, the progression of ALP over time can be decomposed into the contribution of technological change, measured here by TLP, and a second term that captures the effect of the increasing capital-labour ratio (movement along the isoquant) that goes with the increase in KIP. In a way, this somewhat alien term stands for the lack of recognition of the role of capital in production when computing ALP: it is what it takes to convert a total productivity measure (TLP) back to a partial one (ALP). Expression (18) shows that since KIP tends to increase over time ALP will typically exceed TLP.

Our discussion could easily be replicated by focusing on capital rather than on labour. All we need to do is to interchange the subscripts K and L, and one ends up with a *total capital productivity* (TKP) measure, one that is in the spirit of Solow-neutral technological change, with the numerator being output net of the input of labour services. Thus, in analogy to (17), we would obtain:[20]

$$
\begin{aligned}
TKP_{t,t-1} &\simeq Q_{Y,t,t-1}^{1/\bar{s}_{K,t,t-1}} X_{L,t,t-1}^{-\bar{s}_{L,t,t-1}/\bar{s}_{K,t,t-1}} X_{K,t,t-1}^{-1} \\
&= \left(Q_{Y,t,t-1} X_{L,t,t-1}^{-\bar{s}_{L,t,t-1}} X_{K,t,t-1}^{-\bar{s}_{K,t,t-1}} \right)^{1/\bar{s}_{K,t,t-1}} \\
&= TFP_{t,t-1}^{1/\bar{s}_{K,t,t-1}}
\end{aligned} \tag{19}
$$

Given that $\bar{s}_{K,t,t-1} < \bar{s}_{L,t,t-1}$ typically, one would normally expect $TKP_{t,t-1}$ to exceed both $TLP_{t,t-1}$ and $TFP_{t,t-1}$, as long as the latter is indeed greater than one. This might come as a surprise in view of our earlier comment that AKP growth is likely to be negative since the denominator (capital) then tends to grow more rapidly than the numerator (output). The situation is different with TKP growth, though, for now it is not just the denominator that may grow rapidly, but the numerator – being magnified, so to speak – will too, and, as long as the fraction is greater than unity, the latter effect will dominate. This Solow-like index would be particularly relevant in the context of an economic model treating labour as a variable input, in the presence of unemployment for instance,[21] or if the focus is on the return to capital as in Lawrence, Diewert, and Fox (2006).

Total Labour Productivity: A Dual Approach

There is another way to conduct our analysis. Let $\Lambda_{t,t-1}$ be the growth factor

20 This TKP measure would be equivalent to what Lawrence, Diewert, and Fox (2006) label a capital TFP measure.

21 See Kohli (1983), for instance.

of the wage bill. From (10) we can write:

$$\Lambda_{t,t-1} = \frac{\Lambda_t}{\Lambda_{t-1}} = W_{L,t,t-1} \cdot X_{L,t,t-1} \quad (20)$$

We next define the *implicit* price index of net output:

$$\tilde{P}_{\Lambda,t,t-1} = \frac{\Lambda_{t,t-1}}{Q_{\Lambda,t,t-1}}, \quad (21)$$

which has the implicit (or indirect) Törnqvist form. Note that this index is not identical to the *direct* Törnqvist price index, which, in analogy to (12), would be given by:

$$P_{\Lambda,t,t-1} = P_{Y,t,t-1}^{\bar{\lambda}_{Y,t,t-1}} \cdot W_{K,t,t-1}^{-\bar{\lambda}_{K,t,t-1}} \quad (22)$$

This is due to the fact that the Törnqvist does not satisfy the factor reversal test. This is a minor flaw, for it is well known that direct and implicit Törnqvist are typically numerically very close to one another (Diewert 1976; 1978). In what follows, we will therefore use either one as a close approximation for the other.[22]

It then immediately follows from (13), (20) and (21) that TLP can be expressed in the dual price space:

$$TLP_{t,t-1} = \frac{W_{L,t,t-1}}{\tilde{P}_{\Lambda,t,t-1}} \quad (23)$$

That is, TLP can also be thought of as the increase in real wages measured in terms of net output, a finding that is not really surprising in view of a similar result of Jorgenson and Griliches (1967) with regard to TFP:

$$TFP_{t,t-1} = \frac{W_{t,t-1}}{\tilde{P}_{Y,t,t-1}} \quad (24)$$

where $\tilde{P}_{Y,t,t-1}$ is the implicit GDP price deflator. Interestingly enough, an expression somewhat similar to (23) appears in Domar's (1962) review of Kendrick (1961), but he dismisses it as "merely an index of the real wage rate".[23]

The point, though, is that $\tilde{P}_{\Lambda,t,t-1}$ — the denominator in (23) — is not the price of gross output, but the price of the value added by labour, so that the ratio (23) does indeed provide a measure of technological change.

Whereas TFP and TLP can be expressed in terms of price changes, things are not quite that obvious when it comes to ALP since, by totally excluding capital, it is a partial productivity index. Nonetheless, substituting prices for quantities and vice versa, the mirror image of ALP that emerges is the real wage rate, i.e. the *marginal labour productivity* (MLP) index; the *marginal capital productivity* (MKP) index can be similarly defined. We thus

22 Alternatively, in order not to have to make any arbitrary choices, we could use a *symmetric* Törnqvist index defined as their geometric mean:

$$\hat{P}_{\Lambda,t,t-1} = \left(P_{\Lambda,t,t-1}\,\tilde{P}_{\Lambda,t,t-1}\right)^{1/2} = \left[\left(\frac{P_{Y,t,t-1}}{Q_{Y,t,t-1}}\right)^{\bar{\lambda}_{Y,t,t-1}} \cdot \left(\frac{W_{K,t,t-1}}{X_{K,t,t-1}}\right)^{-\bar{\lambda}_{K,t,t-1}} \cdot \Lambda_{t,t-1}\right]^{1/2}$$

and similarly on the quantity side. The resulting indices then satisfy the factor reversal test. See Kohli (2025).

23 See Domar (1962: 604), expression (12).

obtain:

$$\begin{aligned} MLP_{t,t-1} &= \frac{W_{L,t,t-1}}{P_{Y,t,t-1}} \\ MKP_{t,t-1} &= \frac{W_{K,t,t-1}}{P_{Y,t,t-1}} \end{aligned} \tag{25}$$

The links between MLP, TFP, and TLP can also be examined in the dual price space. In analogy to (9) we find:

$$\begin{aligned} MLP_{t,t-1} &= \frac{W_{L,t,t-1}}{P_{Y,t,t-1}} \\ &= \frac{W_{K,t,t-1}^{\bar{s}_{K,t,t-1}} W_{L,t,t-1}^{\bar{s}_{L,t,t-1}}}{P_{Y,t,t-1}} \cdot \left(\frac{W_{L,t,t-1}}{W_{K,t,t-1}}\right)^{\bar{s}_{K,t,t-1}} \\ &= TFP_{t,t-1} \left(\frac{W_{L,t,t-1}}{W_{K,t,t-1}}\right)^{\bar{s}_{K,t,t-1}} \end{aligned} \tag{26}$$

Thus, the growth in MLP will both reflect technological progress, as measured by the increase in TFP, and technical change, i.e. changes in the slopes of the isoquants at the production point. As for the link between TLP and TFP, following the same steps as in (17), we find:

$$\begin{aligned} TLP_{t,t-1} &= \frac{W_{L,t,t-1}}{\tilde{P}_{\Lambda,t,t-1}} \simeq \frac{W_{L,t,t-1}}{P_{\Lambda,t,t-1}} \\ &= \frac{W_{L,t,t-1}}{P_{Y,t,t-1}^{\bar{\lambda}_{Y,t-1}} W_{K,t,t-1}^{-\bar{\lambda}_{K,t,t-1}}} \\ &\simeq \left(W_{L,t,t-1}^{\bar{s}_{L,t,t-1}} P_{Y,t,t-1}^{-1} W_{K,t,t-1}^{\bar{s}_{K,t,t-1}}\right)^{1/\bar{s}_{L,t,t-1}} \\ &= TFP_{t,t-1}^{1/\bar{s}_{L,t,t-1}} \end{aligned} \tag{27}$$

The mirror image of (18), finally, yields the relation between MLP and TLP:

$$\begin{aligned} MLP_{t,t-1} &= \frac{W_{L,t,t-1}}{P_{Y,t,t-1}} \\ &= \frac{W_{L,t,t-1}}{P_{Y,t,t-1}^{1+\bar{\lambda}_{K,t,t-1}} P_{Y,t,t-1}^{-\bar{\lambda}_{K,t,t-1}} W_{K,t,t-1}^{-\bar{\lambda}_{K,t,t-1}} W_{K,t,t-1}^{\bar{\lambda}_{K,t,t-1}}} \\ &= \frac{W_{L,t,t-1}}{P_{\Lambda,t,t-1}} \cdot \left(\frac{W_{K,t,t-1}}{P_{Y,t,t-1}}\right)^{-\bar{\lambda}_{K,t,t-1}} \\ &= TLP_{t,t-1} \cdot MKP_{t,t-1}^{-\bar{\lambda}_{K,t,t-1}} \end{aligned} \tag{28}$$

Thus, while TLP is a complete measure of technological change from a labour perspective, MLP, which also includes the effect of technical change (resulting from the change in the capital-labour ratio), is a hybrid productivity measure that is "merely an index of the real wage rate", and thus just a *partial productivity measure* to use Domar's own terminology.

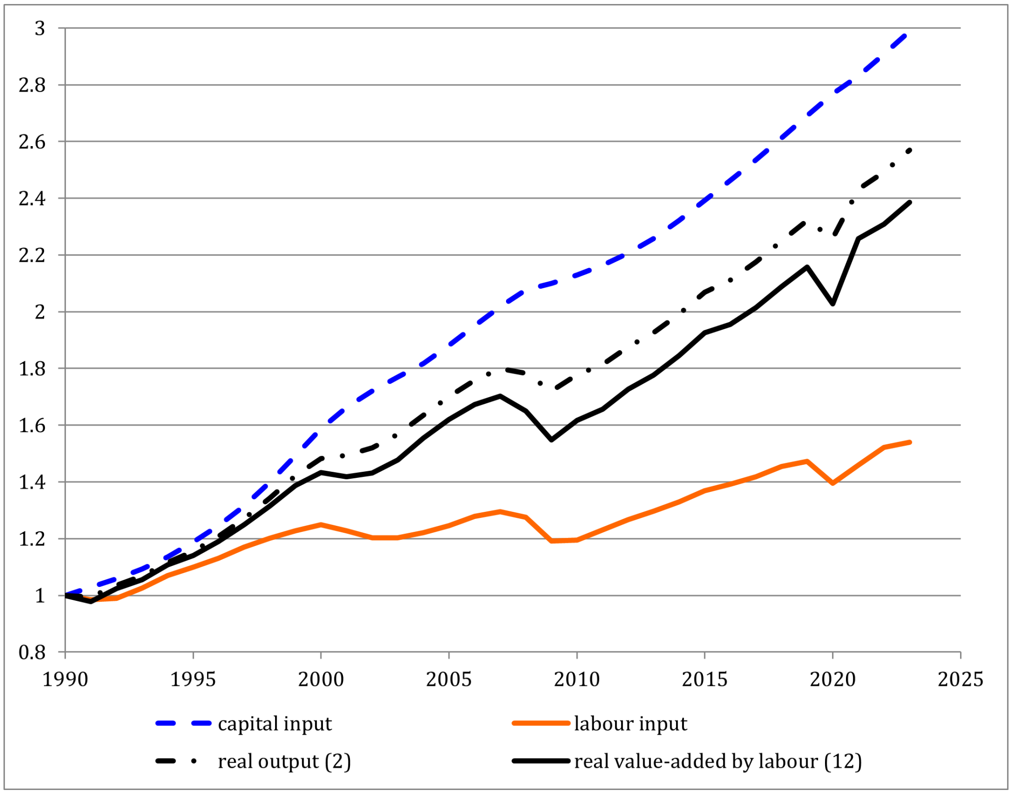
Numerical Illustration

We will take the case of the U.S. private nonfarm business sector, 1990–2023, as an illustration. The data are from the Bureau of Labor Statistics (BLS, 2017; 2024). We basically require series for real output ("real value-added output"), as well as for the capital and labour input quantities. These are readily available from the BLS in index form.[24] We further require current-value data on labour compensation and capital costs in order to be able to measure input shares; these are also available from the BLS.

Chart 1 shows the paths of the four quantity variables of interest: capital input ($x_{K,t}$), labour input ($x_{L,t}$), real out-

24 Note that there is no need to worry about intermediate inputs since domestic intermediate inputs cancel out at the aggregate level, whereas imports are already netted out from domestic output.

Chart 1: Input and Output Data U.S. Private Nonfarm Business Sector, 1990-2023



Source: Computed by the author on the basis of BLS (2024)

put ($q_{Y,t}$), and real output net of capital services ($q_{\Lambda,t}$) obtained by compounding $Q_{\Lambda,t,t-1}$ as given by (12). Real output of the U.S. private nonfarm business sector is found to have increased by 156.9 per cent over the 33-year period (corresponding to an average annual rate of about 2.9 per cent). The growth was not smooth, though: two dips in the growth path are clearly visible, in 2009 on the aftermath of the financial crisis, and in 2020 following Covid-19. Labour services increased by 54.0 per cent (a 1.3 per cent average yearly increase) over these three decades. Capital services nearly trebled over the same time, increasing by 198.8 per cent (a 3.4 per cent average yearly growth rate). As for real value-added by labour, i.e. real net output treating capital services as an intermediate input, the data reveal an increase of 138.6 per cent, which corresponds to an annual increase of 2.7 per cent.

Our first three measures of productivity are reported in Table 1, and depicted graphically in Chart 2.[25] Not surprisingly,

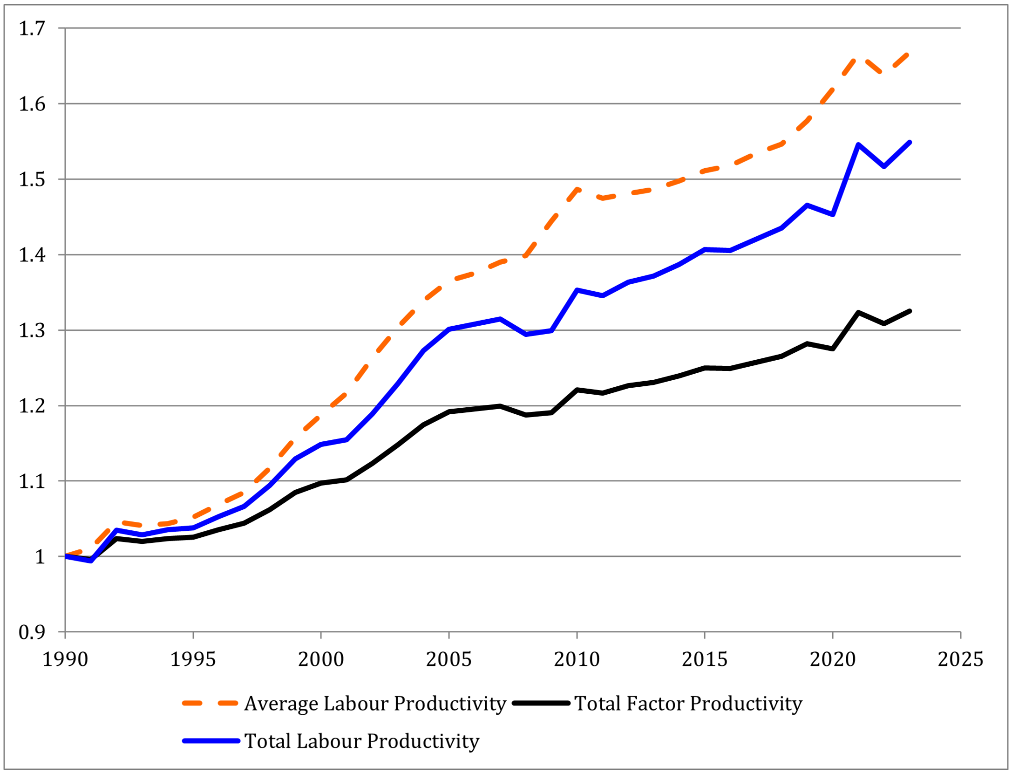
25 It must be noted that our measure of ALP differs from the "labor productivity" index published by the BLS since the BLS uses the number of hours worked—rather than its own measure of labour input—as the denominator, thereby ignoring its labour composition index; the BLS's "labor productivity" index, that records a 94.8 per cent increase over the sample period, is thus all the more a partial index. If one really wanted to focus on hours worked exclusively when assessing TLP, one should use as the numerator a measure of real value added net of the contribution of capital services and net of the contribution of the labour composition index.

Table 1: Average Labour, Average Capital, Total Factor, Total Labour, and Total Capital Productivity Measures: U.S. Private Nonfarm Business Sector, 1990–2023

	Average Labour Productivity	Average Capital Productivity	Total Factor Productivity	Total Labour Productivity	Total Capital Productivity
Year	$ALP_{t,t-1}$ (4)	$AKP_{t,t-1}$ (4)	$TFP_{t,t-1}$ (8)	$TLP_{t,t-1}$ (13)	$TKP_{t,t-1}$ (19)
1991	1.0108	0.9664	0.9962	0.9944	0.9883
1992	1.0350	1.0119	1.0277	1.0407	1.0905
1993	0.9949	0.9986	0.9961	0.9942	0.9876
1994	1.0027	1.0071	1.0042	1.0062	1.0126
1995	1.0079	0.9898	1.0017	1.0026	1.0051
1996	1.0161	0.9964	1.0093	1.0142	1.0277
1997	1.0151	0.9958	1.0085	1.0129	1.0253
1998	1.0293	0.9936	1.0173	1.0260	1.0531
1999	1.0359	0.9927	1.0215	1.0322	1.0664
2000	1.0265	0.9809	1.0113	1.0168	1.0347
2001	1.0245	0.9613	1.0038	1.0055	1.0117
2002	1.0379	0.9837	1.0198	1.0296	1.0620
2003	1.0323	1.0025	1.0221	1.0336	1.0665
2004	1.0269	1.0157	1.0230	1.0354	1.0677
2005	1.0201	1.0046	1.0146	1.0227	1.0413
2006	1.0069	0.9972	1.0034	1.0053	1.0093
2007	1.0108	0.9900	1.0032	1.0050	1.0088
2008	1.0063	0.9626	0.9900	0.9843	0.9730
2009	1.0329	0.9536	1.0026	1.0042	1.0071
2010	1.0291	1.0193	1.0253	1.0412	1.0677
2011	0.9919	1.0044	0.9967	0.9946	0.9915
2012	1.0043	1.0141	1.0081	1.0132	1.0210
2013	1.0036	1.0032	1.0035	1.0057	1.0090
2014	1.0076	1.0058	1.0069	1.0114	1.0177
2015	1.0088	1.0083	1.0086	1.0140	1.0223
2016	1.0047	0.9910	0.9994	0.9991	0.9985
2017	1.0104	1.0006	1.0066	1.0108	1.0174
2018	1.0080	1.0040	1.0064	1.0105	1.0168
2019	1.0199	1.0016	1.0128	1.0210	1.0334
2020	1.0269	0.9462	0.9947	0.9914	0.9865
2021	1.0286	1.0525	1.0380	1.0640	1.0983
2022	0.9831	0.9972	0.9888	0.9812	0.9728
2023	1.0186	1.0036	1.0124	1.0211	1.0303
1990–2023	**1.6677**	**0.8597**	**1.3248**	**1.5487**	**2.2174**
Annual mean	**1.0156**	**0.9954**	**1.0086**	**1.0133**	**1.0244**

Note: All reported numbers are growth factors.
Source: Computed by the author on the basis of BLS (2024)

Chart 2: Average Labour, Total Factor, and Total Labour Productivity Measures Cumulated values, U.S. Private Nonfarm Business Sector, 1990-2023



Source: Computed by the author on the basis of BLS (2024)

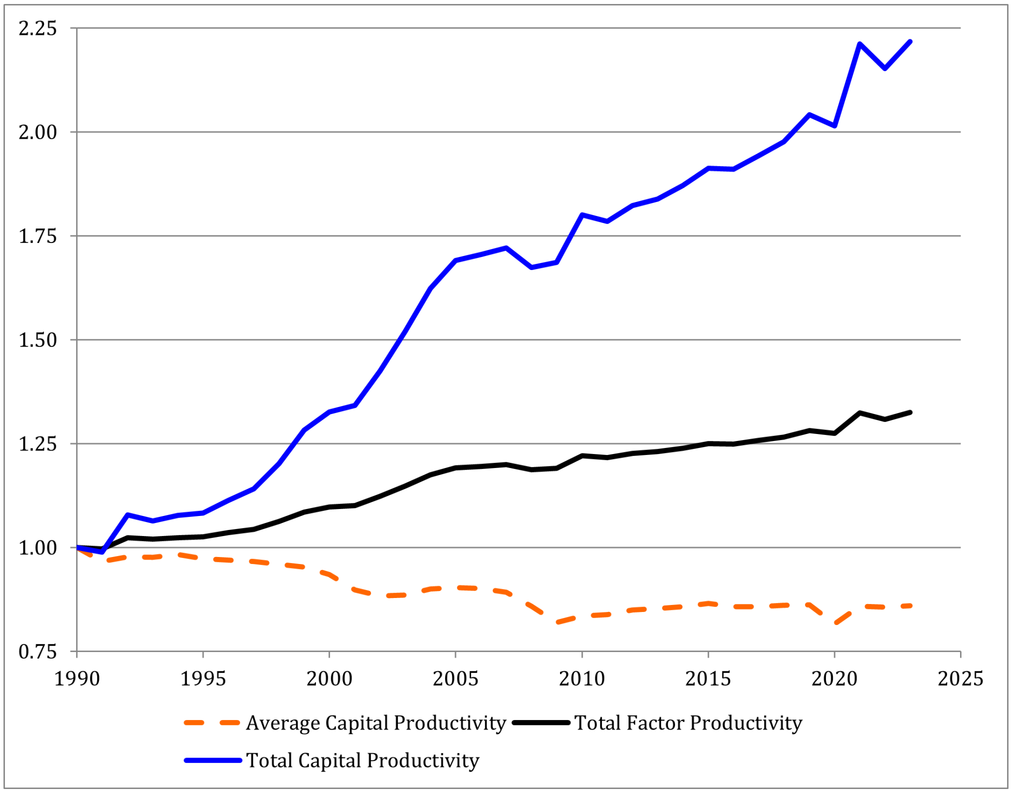
the lower growth rate of the labour input—compared to capital, and thus to output—implies a rather healthy estimate of ALP growth at about 66.8 per cent (1.6 per cent per annum) over the sample period; see Table 1, column 1. As we have come to expect, not all of this is technological progress, since the capital-labour ratio has increased substantially over the sample period, by a total of 94.0 per cent (2.0 per cent per year).[26] If the effect of the implied technical change shown by the last term on the right-hand side of expression (9) is netted out (it adds up to about 25.9 per cent, i.e. 0.7 per cent yearly), or if one simply uses expression (8) directly, then obtains a TFP increase of 32.5 per cent over the sample period, which amounts to close to 0.9 per cent annually; see Table 1, column 3.

If one wants to use labour as a benchmark, all technological progress may be allocated to it, but only after having netted out the contribution of capital to production. In that case, as shown by (17), the effect is magnified by a factor of $1/\bar{s}_L$ to yield a total TLP increase of nearly 54.9 per cent over the period, i.e. about 1.3 per cent annually as shown in Table 1, column 4.

The paths of our first three measures of productivity growth are shown in Chart 2. One can see that TLP dominates TFP,

26 For its measure of capital-labour intensity too, the BLS uses the number of hours worked as the denominator, whereas we use its labour input measure instead.

Chart 3: Average Capital, Total Factor, and Total Capital Productivity Measures Cumulated values, U.S. Private Nonfarm Business Sector, 1990-2023



Source: Computed by the author on the basis of BLS (2024)

while ALP is found to progress even more rapidly. One would normally expect capital to grow more rapidly than labour: TFP growth should therefore fall short of ALP growth. This is indeed what expression (9) predicts. Nonetheless, there are a few observations in our sample where this relationship is reversed, in particular in 2021–2022 when employment recovered strongly after a dramatic fall in 2020 as a consequence of Covid-19, thus leading to a rather exceptional reduction in the capital-labour ratio. As predicted by (17), TLP exceeds TFP whenever technological progress (i.e. Domar's Residual) is positive, which happened in the vast majority of the observations. As for the relationship between ALP and TLP, we find that the former exceeds the latter in just over half the observations. In the remaining cases, the capital intensity of production actually fell somewhat, thus reversing the inequality in accordance with (18).

For the sake of completeness, we also report values of AKP and TKP—the Solow-like index of technological progress—as given by (19). These are shown in columns 2 and 5 of Table 1. As expected, AKP is mostly falling over time, at an average rate of about 0.5 per cent for a total decline of 14.0 per cent, whereas TKP is increasing at an average yearly rate of 2.4 per cent, thus more than doubling (a 121.7 per cent rise) over the sample period. This substantial increase can be explained by the magnification effect relative to TFP due to the relatively smaller capital share as shown by (19). The paths of AKP, TFP, and TKP are shown in Chart 3. While the divergence between the partial and total productivity measures was already evident in Chart 2 for labour, the contrast is even more striking for capital, with one measure declining steadily and the other one increasing sharply. This demonstrates that productivity measures should not be defined casually, but rather with a definite framework in mind.

Concluding Comments

As stressed throughout this article, ALP is not suitable as a measure of technological change since it is a partial productivity index that totally neglects the role of capital, not so much when it comes to the shifts in the technology, but much more importantly in production altogether. This is if one considers the national income identity: any attempt to express a link between $q_{Y,t}$ and $x_{L,t}$ that excludes $x_{K,t}$, on either side of the identity, is incomplete. Given that it is perfectly appropriate to focus on labour (the denominator in that case), one must conclude that it is the numerator that is faulty. One might object that this is untrue, for, as shown by (9), ALP does take capital into account, even twice so: first explicitly in the last right-hand terms of (9), and a second time implicitly in the denominator as an element of TFP. That is exactly the point: one cancels the other one out.

It would be a simple matter for statistical agencies to publish series on TLP (and TKP) in the future: all the necessary data are readily available. Thus, computing $Q_{\Lambda,t,t-1}$ and TLP with the help of (12) and (13) is no more difficult than deriving $X_{t,t-1}$ and TFP using (7) and (8). Labour productivity measures are often used for international comparisons, not least by the International Labour Organization (2025). It would then be of advantage to compare "pure" labour productivity series, i.e. data that are not tainted by the hidden influence of capital accumulation.

One could of course also define labour productivity in terms of *net domestic product* (NDP) as opposed to GDP. The measure of ALP would be directly impacted if the numerator were real NDP rather than real GDP, but what ultimately matters is the growth in productivity rather than its level, so it is not possible to come to definite conclusions in the absence of information about the rate at which fixed capital is consumed. The measure of TLP would presumably be less affected, since only the weights $\lambda_{Y,t}$ and $\lambda_{K,t}$, as defined in (11) would be somewhat reduced.

The difference between ALP and TLP also is particularly meaningful if the country (or the firm) is heavily indebted. Although the ALP and TLP measures would not be affected if part or all of the capital income were due to foreign investors, TLP would be much more relevant than ALP from a national—as opposed to domestic—income perspective.

A further point that speaks in favour of TLP is that this measure of labour productivity is more directly related to real wages than is ALP. The passage from TLP to MLP, as it can be seen by comparing (25) with (23), is really quite simple: it is just a matter of replacing one price deflator ($P_{\Lambda,t,t-1}$) by another one ($P_{Y,t,t-1}$). This contiguity is also supported by the data. Thus, while the BLS figures indicate that real wages increased by a factor of 1.45 over the sample period, TLP increased by a factor of 1.55, whereas ALP was multiplied by 1.67 (see Table 1). The notion that ALP- and MLP-growth should be equal is inherited from the common reference to Cobb-Douglas production functions that restrict the elasticity of substitution between labour and capital to unity and thus imply constant factor shares, whereas the reality is quite different, with the U.S. capital share having tended to increase over the sample period.

Yet another area where the concept of TLP could find a useful application is when it comes to unit labour costs (ULC). Many statistical agencies, including the BLS, publish ULC measures. ULC, defined as total labour compensation divided by real output, or, equivalently, the nominal wage divided by ALP, are often used in international competitiveness comparisons (OECD, 2025). It would certainly make sense, in such studies, to substitute TLP for ALP as the denominator in order to obtain a clean ULC index, a measure of the cost of the real value added by labour without having the unaccounted-for influence of capital.

Naturally, TLP is also relevant at the sector, the industry, and even the firm level. The real value added by labour can be calculated as described above, after having netted out the contribution of capital and, if relevant, of intermediate inputs such as energy, materials, and purchased services.

To sum up, our purpose in this article is not to advocate the rejection of ALP as a measure of productivity growth. It is a descriptive – rather than analytical – statistic that is informative about the state of the economy and its historical evolution. In a way, just like it is true for capital, its inverse – in this case the labour intensity of production – is just as informative. In any case, it is important to remind users that ALP is a hybrid measure that combines the effects of shifts in the production possibilities frontier (e.g. shifts of the isoquants related to a production function) and movements along that frontier (e.g. movement along an isoquant). Expanded knowledge and improvements in the technology are not the same as plain capital accumulation.

This does not mean, however, that labour should not be used as a benchmark. Quite the contrary: we have listed in the introduction a number of reasons why a labour productivity measure is appealing. This is all the more true if technological change tends to be mostly labour augmenting: a Harrod-like measure such as TLP is therefore particularly appropriate. TFP as a measure of technological change, of course, retains all its validity and its importance, but one must realize that it implicitly describes a Hicks-neutral type of technological change.

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