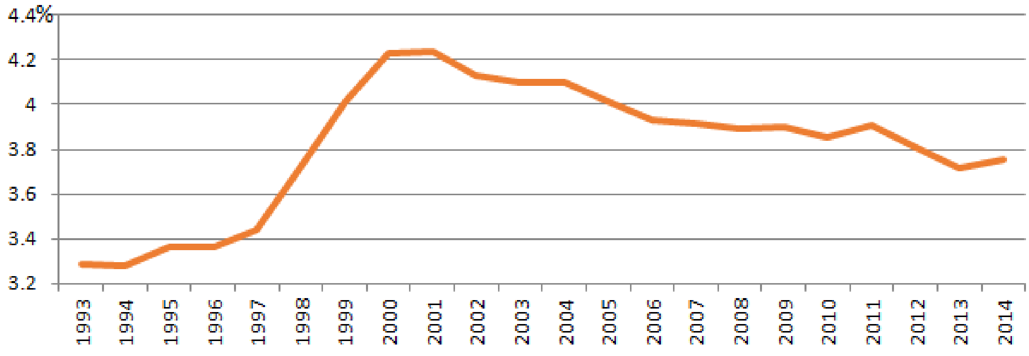
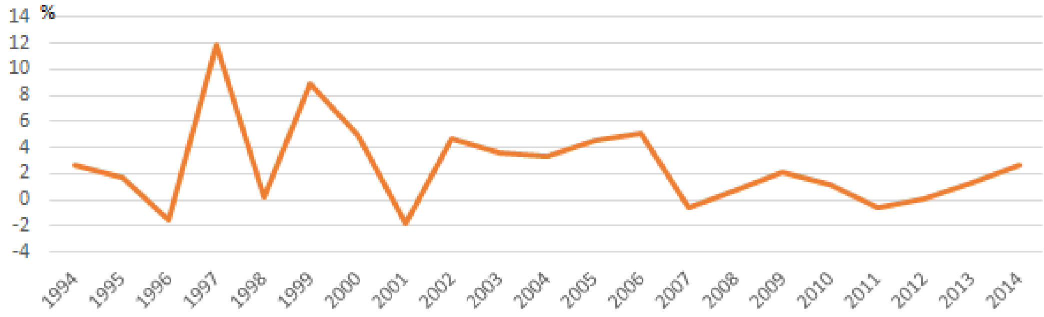
Appendix A: Supplementary Data and Analysis

Chart A1: ICT Sector Share of Total Hours Worked (annual data)



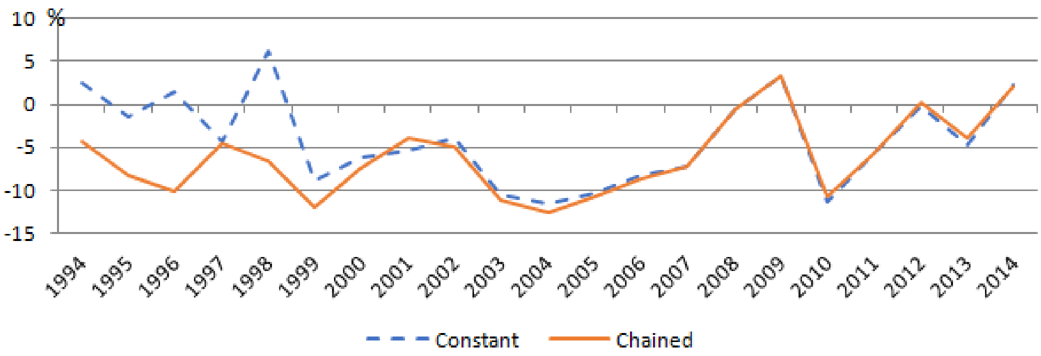
Source: Statistics Canada.

Chart A2: ICT Sector Share of Total Hours Worked (annual data)



Source: Statistics Canada.

Chart A3: Computer and Telecommunication Price Growth (annual data)



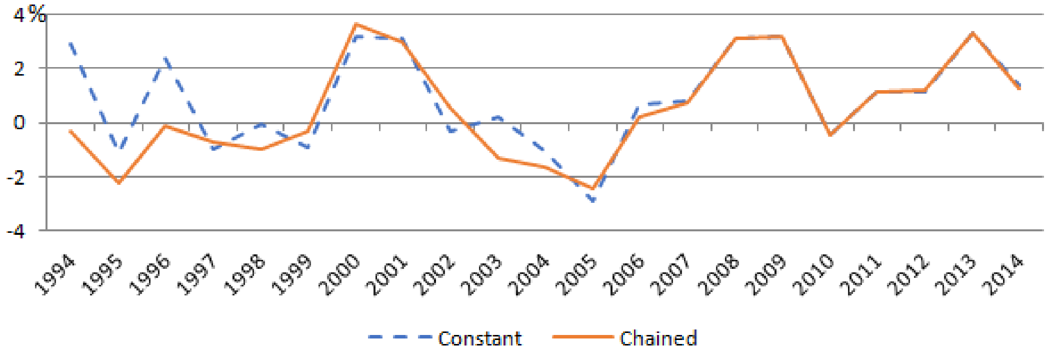
Source: Statistics Canada.

Table A1: Price Growth (constant dollars)

	1993-2003	2003-2014
Non-ICT sector GDP deflator growth	**1.99**	**2.38**
ICT investment price growth	**-1.09**	**-2.03**
Computer hardware	-8.13	-6.25
Telecommunications equipment*	-0.53	-2.69
Software	0.85	0.93

Source: Statistics Canada and authors calculations.
Notes: Indicates data received from Statistics Canada through special request.

Chart A4: Software Price Growth (annual data)



Source: Statistics Canada.

Table A2: Shift-share Results

	1993-2003	2003-2014
Labour productivity growth	1.59	1.03
ICT sector contribution (p.p.p.y)	**0.14**	**0.08**
Reallocation	0.01	-0.01
Within-sector	0.14	0.09
All else	1.45	0.95

Source: Statistics Canada and authors calculations.
Note that the shift-share was performed using the CSLS method as described by de Avillez (2012), where reallocation is the sum of the reallocation growth effect and the reallocation level effect. The exercise uses constant 2007 dollars. The labour productivity level for the ICT sector was $48 (GDP per hour worked) in the 19932003 compared with $44 in the non-ICT sector. In the 20032014 period, the ICT sector had a productivity level of $64 while the non-ICT sector was $50.

Appendix B: Relaxing the Assumption of Identical Production Functions in the ICT and non-ICT Sectors

In this Appendix, we examine the implications, in an approach " la Oulton (2012)", of relaxing the assumption that the production functions of the ICT and non-ICT sectors are identical. We show that this gives expressions that do not capture the contribution of ICT to productivity growth. We also propose an alternative approach, "CUPP", more consistent with the contribution of ICT in a world with different production functions.

We start with the two production functions described in Section 2.2:

$$y_N = B_N (k_N^N)^{\gamma} (k_N^T)^{\sigma} (h_N)^{1-\gamma-\sigma} \tag{A1}$$
$$0 < \gamma, \sigma < 1; \gamma + \sigma < 1$$

$$y_T = B_T (k_T^N)^{\theta} (k_T^T)^{\eta} (h_T)^{1-\theta-\eta} \tag{A2}$$
$$0 < \theta, \eta < 1; \theta + \eta < 1$$

In equations (A1) and (A2), the variables are as defined in Section 2.2.

We can obtain expressions for the growth rates of equations (A1) and (A2) by taking total derivatives:

$$\dot{y}_N = \mu_N + \gamma \dot{k}_N^N + \sigma \dot{k}_N^T + (1-\gamma-\sigma)\dot{h}_N \tag{A3}$$

$$\dot{y}_T = \mu_T + \gamma \dot{k}_T^N + \sigma \dot{k}_T^T + (1-\gamma-\sigma)\dot{h}_T \tag{A4}$$

Again, in equations (A3) and (A4), the variables are as defined in Section 2.2.

In the non-ICT sector, the use cost can be approximated by $(r+\delta_N-\dot{p}_N)p_N$, r is the real interest rate and δ_N is the rate of depreciation of non-ICT capital. Normalizing to non-ICT prices and setting the marginal product of capital equal to the user cost, we have

$$(r+\delta_N) = \gamma \frac{y_N}{k_N^N}$$
$$(r+\delta_T-\dot{p})p = \sigma \frac{y_N}{k_N^T}$$

where $p = \frac{p_T}{p_N}$. Given that in the steady-state interest rates and depreciation rates are fixed, we have that

$$\bar{\dot{y}} = \bar{\dot{k}}_N^N = \bar{\dot{k}}_N^T + \dot{p} \tag{A5}$$

Similarly, $(r+\delta_T+\dot{p}) = \eta \frac{y_T}{k_T^T}$ implies that $\bar{\dot{y}}_T = \bar{\dot{k}}_T^T$.

Assuming that in a steady-state $\sigma \frac{y_N}{k_N^T} = \eta \frac{y_T}{k_T^T} p$, implies that

$$\bar{\dot{y}}_N - \bar{\dot{k}}_N^T = \bar{\dot{y}}_T - \bar{\dot{k}}_T^T + \dot{p} \tag{A6}$$

Rearranging (A6), we get that

$$\bar{\dot{y}}_T = \bar{\dot{y}}_N + (\bar{\dot{k}}_T^T - \bar{\dot{k}}_N^T) - \dot{p} \tag{A7}$$

Substituting equations (A5) into (A3) we obtain an expression for labour productivity growth in the non-ICT sector

$$\bar{\dot{y}}_N = \mu_N + \gamma \bar{\dot{y}}_N + \sigma(\bar{\dot{y}}_N - \dot{p}) + \bar{\dot{h}}(1-\gamma-\sigma) \tag{A8}$$

In the steady-state, we assume that

aggregate labour productivity growth can be expressed as a weighted average of the two sectors' productivity growth rates, i.e.

$$\bar{\dot{y}} = (1-\bar{q}_T)\bar{\dot{y}}_N + \bar{q}_T\bar{\dot{y}}_T = \bar{\dot{y}}_N + \bar{q}_T(\bar{\dot{y}}_T - \bar{\dot{y}}_N). \tag{A9}$$

Oulton's original model assumes instead that steady-state labour productivity growth is the sum of labour productivity growth in the two sectors weighted by their respective nominal output share. We use the hours share because when we assume different production functions, aggregate nominal output is no longer constant. But we still assume that hours growth is the same in the ICT and the non-ICT sector in the steady-state.

Substituting equation (A7) into (A9)

$$\bar{\dot{y}} = \bar{\dot{y}}_N + \bar{q}_T([\bar{\dot{k}}_T^T - \bar{\dot{k}}_N^T] - \dot{p}). \tag{A10}$$

Re-arranging equation (A8) gives

$$\bar{\dot{y}}_N = \frac{\mu_N}{(1-\gamma-\sigma)} + \bar{\dot{h}} + \frac{\sigma}{(1-\gamma-\sigma)}(-\dot{p}) \tag{A11}$$

Substituting (A11) into (A10) gives

$$\begin{aligned}\bar{\dot{y}} = &\frac{\mu_N}{(1-\gamma-\sigma)} + \bar{\dot{h}} + \\ &\frac{\sigma}{(1-\gamma-\sigma)}(-\dot{p}) + \\ &\bar{q}_T([\bar{\dot{k}}_T^T - \bar{\dot{k}}_N^T] - \dot{p})\end{aligned} \tag{A12}$$

where, if we followed Oulton's approach, the last two terms would reflect the contribution of ICT to labour productivity, as is expressed in equation (10) of Section 2.4. But these two terms are clearly different from the solution to the original Oulton model. This is made clear by the presence of $[\bar{\dot{k}}_T^T - \bar{\dot{k}}_N^T]$. Why would stronger ICT capital investment in the non-ICT sector, as reflected in $\bar{\dot{k}}_T^T$, be negative for productivity?

We therefore take a different approach. Substituting equation (A11) in equation (A9) gives

$$\begin{aligned}\bar{\dot{y}} = \bar{q}_T\bar{\dot{y}}_T + (1-\bar{T})\frac{\sigma}{(1-\gamma-\sigma)}(-\dot{p}) + \\ (1-\bar{q}_T)[\frac{\mu_N}{(1-\gamma-\sigma)} + \bar{\dot{h}}]\end{aligned} \tag{A13}$$

The first two terms on the right-hand side of equation (A12) measure the contribution of ICT to aggregate labour productivity as expressed in equation (9) and discussed in Section 2.4.

Appendix C: Information on Constructed or Custom Variables

In this appendix, we expand on Section 3 to discuss several data issues that we experience when estimating the models used in this paper with Canadian data.

Gross Domestic Product

As mentioned in Section 3, we use three NAICS codes to approximate the ICT sector: NAICS 334, 51, and 5415.

Whenever real GDP is required, we use 2007 constant dollars, as opposed to a Divisia index, in order to allow for the summation of real goods and services across sectors.

The ICT GDP series that is created concatenates data from three different CANSIM tables. GDP for NAICS 5415 is unavailable pre-1997, and so from 1993-1996 we backcast under the assumption that the growth rate of this sectors output is equal to that of NAICS 54.

Capital, Capital Cost, and the Labour Share

ICT capital is defined as computer hardware, telecommunication equipment, and software. We received the disaggregation from Statistics Canada by special request for telecommunication and computer hardware for the total economy, as well as for the NAICS codes 51 and 334. Computer system design and related services, however, did not have capital data as it was aggregated at the two-digit level. Therefore, to construct a series for ICT capital stock and ICT capital cost we first assume that, like the rest of the economy, this sector has constant returns to scale. Given this assumption and that data exists for nominal GDP and labour compensation, we are able to obtain total capital cost for the sector. Next, using data acquired from Statistics Canadas MFP program, we are able to break down the share of the cost of capital services that is ICT up until 2008, which we extend to 2014 using the shares from NAICS 54.[26] To get the real ICT capital in the sector, we calculate a ICT user cost by dividing the ICT capital cost in the total economy by real capital, and then applying that to NAICS 5415 to get an estimate of the real ICT capital in the sector.

ICT capital cost can be calculated using a simple Jorgenson user cost as shown in Appendix B. To calculate the capital cost for the total economy, however, we decided to use the capital cost series from Statistics Canadas MFP program for the business sector, and apply the ratio of ICT capital cost out of total capital cost to the aggregate economy after assuming constant returns to scale (we also tested the robustness of our results to this by using ratios from Statistics Canadas MFP program with very similar results). We then disaggregated the ICT capital cost by type of capital

26 We also test using the average share of ICT from 2004-2008, as well as using the shares from the other two ICT- producing sector, which yields the same qualitative conclusion.

by using the shares calculated using the Jorgenson formulas.

The constant returns to scale is an important assumption as much of the data we use will depend upon the labour share. We calculate the labour share simply as the labour compensation divided by nominal GDP at basic prices. This gives a slightly lower labour share than is sometimes calculated, for example in Statistics Canadas Productivity Accounts.